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Quantum weakdynamics as an $SU(3)_I$ gauge theory: Grand unification of strong and electroweak interactions

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Abstract. Quantum weakdynamics (QWD) as an $SU(3)_I$ gauge theory with the Θ vacuum term is considered to be the unification of the electroweak interaction as an $SU(2)_\mathrm{L} \times U(1)_Y$ gauge theory. The grand unification of $SU(3)_I \times SU(3)_C$ beyond the standard model $SU(3)_C \times SU(2)_\mathrm{L} \times U(1)_Y$ is established by the group $SU(3)_I$. The grand unified interactions break down to weak and strong interactions at a new grand unification scale, 10^3 GeV, through dynamical spontaneous symmetry breaking (DSSB); the weak and strong coupling constants are the same, $\alpha_i = \alpha_\mathrm{s} \simeq 0.12$, at this scale. DSSB is realized by the condensation of scalar fields, postulated to be spatially longitudinal components of gauge bosons, instead of Higgs particles. Quark and lepton family generation, the Weinberg angle $\sin^2 \theta_\mathrm{W} = 1/4$, and the Cabbibo angle $\sin \theta_\mathrm{C} = 1/4$ are predicted. The electroweak coupling constants are $\alpha_z = \alpha_i/3$, $\alpha_\mathrm{w} = \alpha_i/4$, $\alpha_y = \alpha_i/12$, and $\alpha_e = \alpha_i/16 \simeq 1/137$; there are symmetric isospin interactions.

1 Introduction

The standard model (SM) $SU(3)_C \times SU(2)_\mathrm{L} \times U(1)_Y$ gauge theory [1,2] is quite successful for the phenomenology of elementary particle physics. However, there are still many unexplained phenomena in the SM: for instances, many free parameters, three family generations for leptons and quarks, matter mass generation, the Higgs problem or vacuum problem, dynamical spontaneous symmetry breaking (DSSB) beyond spontaneous symmetry breaking, the neutrino mass problem, etc. In order to resolve these problems, grand unified theories (GUTs) were proposed [3]. Nevertheless, grand unification of the strong and electroweak interactions is not complete, and GUTs also have model dependent problems: the hierarchy problem, proton decay, and the Weinberg angle are problems in $SU(5)$ gauge theory [3]. On the other hand, two important observations can be made. One is that Higgs particles have not been observed yet; this suggests DSSB rather than spontaneous symmetry breaking. The other one is that the experimental strong coupling constant $\alpha_\mathrm{s} \simeq 0.12$ at the energy scale of the intermediate Z^0 vector boson mass [4] and the experimental weak coupling constant $\alpha_\mathrm{w} \simeq 0.03$ at the energy scale of the $W^\pm$ intermediate vector boson mass [5]; the two coupling constants have the same value $\alpha_i = \alpha_\mathrm{s} \simeq 0.12$ around the 10^3 GeV energy scale if an $SU(3)_I$ gauge theory with the coupling constant α_i for the weak force is adopted. These phenomena strongly suggest that all the theoretical problems and experimental facts may thus easily be resolved if quantum weakdynamics (QWD) as an $SU(3)_I$ gauge theory for the weak force is broken down to the Glashow–Weinberg–Salam (GWS) model, the $SU(2)_\mathrm{L} \times U(1)_Y$ theory [1] through DSSB. The aim of this paper accordingly is to propose that QWD as an $SU(3)_I$ gauge theory for the weak force provides hints for the challenging problems and ways to unify the weak and strong force systematically: QWD is analogous to quantum chromodynamics (QCD) as an $SU(3)_C$ gauge theory for the strong force. Many free parameters in the SM must be understood in the context of the grand unification scheme. More precisely, QWD as an $SU(3)_I$ gauge theory is proposed to be the unification of the $SU(2)_\mathrm{L} \times U(1)_Y$ electroweak theory [1], and the grand unification of QCD and QWD is proposed as the unification of the weak and strong interactions [2,6,7] beyond the SM. The proposed group chain is thus given by $H \supset SU(3)_I \times SU(3)_C$ for grand unification, weak, and strong interactions respectively. The grand unified group H of the group $SU(3)_I \times SU(3)_C$ beyond the SM, $SU(3)_C \times SU(2)_\mathrm{L} \times U(1)_Y$, provides coupling constants $\alpha_\mathrm{w} = \alpha_\mathrm{s} \simeq 0.12$ at a new grand unification scale around 10^3 GeV, which might be the resolution of the hierarchy problem of the conventional grand unification scale 10^{15} GeV [3]. QWD provides plausible explanations for the Weinberg angle, the Cabbibo angle, quark and lepton families, a modification to the Higgs mechanism, fermion mass generation, etc. This scheme can be substantiated through further quantum tests; it gives rise to several predictions such as the relation with inflation, the analogy between weak and strong force, the breaking of

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discrete symmetries, etc. The present work is restricted to the real four dimensions of spacetime without considering supersymmetry or higher dimensions. This work is based on phenomenology below the grand unification scale: the electroweak and strong interactions.

As a step toward the grand unification of fundamental forces or toward the systematic description of the evolution of the universe, a new grand unification scale of the $SU(3)_I \times SU(3)_C$ symmetry around 10^3 GeV is necessary. QWD as an $SU(3)_I$ gauge theory provides the unification of the electroweak interaction being an $SU(2)_L \times U(1)_Y$ gauge theory [1]. This scheme resolves the free parameters in the GWS model. In order to show that electroweak interactions stem from an $SU(3)_I$ gauge theory, the roles of scalar fields, parameterized by spatially longitudinal components of gauge bosons, are emphasized instead of the roles of Higgs particles: unsatisfactory factors of the Higgs mechanism [8] can be overcome in this scheme. The essential point is that the system at high energies experiences the stage of DSSB and the effective coupling constant acquires the dimension of inverse energy squared due to massive gauge bosons through DSSB; the effective coupling constant chain due to massive gauge bosons is $G_H \supset G_F \times G_R$ for grand unification, weak, and strong interactions respectively. The DSSB mechanism is adapted to all the interactions characterized by gauge invariance, the physical vacuum problem, and discrete symmetry breaking; the DSSB mechanism is applied to strong interactions having analogous features [6,7]. The DSSB of local gauge symmetry and global chiral symmetry triggers the $(V + A)$ current anomaly. This study suggests that sextet isospin states in two octets of triplet isospin combinations for QWD can produce the three family generations of leptons and quarks. DSSB is closely related to the breaking of discrete symmetries, parity (P), charge conjugation (C), charge conjugation and parity (CP) and time reversal (T). Photons are regarded as massless gauge bosons, Nambu–Goldstone (NG) bosons [9], appearing during DSSB. The quark or lepton mass is generated as the DSSB of gauge symmetry and discrete symmetries, which is motivated by the parameter Θ representing the surface term. Fermion mass generation introduces the common features of constituent particle mass, the dual Meissner effect, and hyperfine structure. The Θ term plays important roles in the DSSB mechanism of the gauge group and in the quantization of the fermion space and vacuum space.

This paper is organized as follows. Section 2 proposes QWD as an $SU(3)_I$ gauge symmetry and describes the common features of DSSB. Section 3 explicitly shows the generation of the $SU(2)_L \times U(1)_Y$ gauge symmetry from the $SU(3)_I$ gauge symmetry. In Sect. 4, the grand unification of $SU(3)_I \times SU(3)_C$ introduces the unified coupling constant at a new grand unification scale. The fermion mass generation mechanism is addressed as being the result of the breaking of gauge and chiral symmetries in Sect. 5. The Θ constant and quantum numbers are discussed in Sect. 6. Section 7 describes a comparison of QWD, GWS, and GUT. Section 8 is devoted to our conclusions.

2 Quantum weakdynamics as an $SU(3)_I$ gauge theory

The generation of the electroweak $SU(2)_L \times U(1)_Y$ theory from QWD as the $SU(3)_I$ gauge theory has several implications. It suggests the DSSB mechanism initiated by the $(V + A)$ current anomaly and predicts several free parameters such as the Weinberg angle, the Cabbibo angle, isospin and electric charge quantization, a modification to the Higgs mechanism, the three family generations of leptons and quarks, fermion mass generation, etc.

DSSB is one of the underlying principles whose principal application is the electroweak theory; remarkably, this unifies weak interactions with electromagnetic interactions in a single larger gauge theory. Here, the DSSB of the weak force from an $SU(3)_I$ gauge theory to an $SU(2)_L \times U(1)_Y$ electroweak theory, the GWS model [1], is addressed as well as the modification of the Higgs mechanism [8]; the phase transition of the electroweak interactions takes place through the condensation of scalar fields, which are postulated to be spatially longitudinal components of gauge bosons, instead of Higgs particles. This scheme uses dynamical symmetry breaking without having to introduce elementary scalar particles; this idea, which aims to have DSSB with gauge interactions alone, is similar to the technicolor model [10] in this sense. Discrete symmetry violation occurs as the result of DSSB. Photons are regarded as massless gauge bosons [9] indicating DSSB. An analogy of superconductivity is expected as a consequence of the condensation of scalar fields.

In this section, the common characteristics of DSSB are discussed following the introduction of QWD: the introduction of QWD, the Θ vacuum as weak CP violation, DSSB through the condensation of scalar fields, the Fermi weak interaction constant and massive gauge bosons, and the breaking of discrete symmetries are addressed.

2.1 Weak interactions as an $SU(3)_I$ gauge theory

QWD is proposed as an $SU(3)_I$ gauge theory, which is an exact copy of QCD in the form apart from the fermion mass term; triplet isospin sources and octet gauge bosons are introduced. A mass term violating gauge invariance is not included but is generated through DSSB. Natural units with $\hbar = c = k = 1$ are used for convenience throughout this paper unless otherwise specified.

The Dirac Lagrangian density apart from the mass term, $\mathcal{L} = \bar{\psi}\mathrm{i}\gamma^\mu\partial_\mu\psi$, leads to the local $SU(3)_I$ gauge theory invariant under the local transformation $\psi \to \mathrm{e}^{\mathrm{i}\lambda\cdot\mathbf{a}/2}\psi$, where λ are the $SU(3)_I$ generators and $\mathbf{a}$ are spacetime parameters. An $SU(3)_I$ gauge theory for the weak interactions can then be established, and the $SU(3)_I$ gauge invariant Lagrangian density is the same, apart from the mass term, as the Lagrangian density of QCD [2] in the form

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}\mathrm{i}\gamma^\mu D_\mu\psi, \quad (1)$$

where ψ stands for the spinor field and $D_\mu = \partial_\mu - \mathrm{i}g_i G_\mu$ stands for the covariant derivative with the coupling constant g_i. The field strength tensor is given by $F_{\mu\nu} = \partial_\mu G_\nu - \partial_\nu G_\mu - \mathrm{i}g_i[G_\mu, G_\nu]$, where spinors carry gauge fields denoted by $G_\mu = \sum_{a=0}^{8} G_\mu^a \lambda^a/2, a = 0, \cdots, 8$ with generators λ^a. The Gell-Mann matrices satisfy the commutation relation $[\lambda_k, \lambda_l] = 2\mathrm{i}\sum_m c_{klm}\lambda_m$ where c_{klm} are the structure constants of the $SU(3)_I$ group. The fine structure constant α_i in QWD is defined analogously to the fine structure constant $\alpha_e = e^2/4\pi$ in QED: $\alpha_i = g_i^2/4\pi$, where α_i is dimensionless.

Three intrinsic isospin (isotope) charges (A, B, C) form the fundamental representation of the $SU(3)_I$ symmetry group. Fermions are combinations of three particles with triplet isospins which produce a decuplet, two octets, and a singlet gauge bosons; $3\otimes3\otimes3 = 10\oplus8\oplus8\oplus1$. Interactions between fermions may be described by two body interactions. The set of unitary 3×3 matrices with $\det U = 1$ forms the group $SU(3)_I$ whose fundamental representation is a triplet. The eight gauge bosons in the octet are for example constructed by a matrix $\sum_1^8 \lambda_k G^k$:

$$\begin{pmatrix} G_3 + G_8/\sqrt{3} & G_1 - \mathrm{i}G_2 & G_4 - \mathrm{i}G_5 \\ G_1 + \mathrm{i}G_2 & -G_3 + G_8/\sqrt{3} & G_6 - \mathrm{i}G_7 \\ G_4 + \mathrm{i}G_5 & G_6 + \mathrm{i}G_7 & -2G_8/\sqrt{3} \end{pmatrix}, \quad (2)$$

where the two diagonal gauge bosons are $G_3 = (A\bar{A} - B\bar{B})/2^{1/2}$ and $G_8 = (A\bar{A} + B\bar{B} - 2C\bar{C})/6^{1/2}$.

The six off-diagonal gauge bosons are accordingly represented by

$$\begin{aligned} A\bar{B} &= (G_1 - \mathrm{i}G_2)/\sqrt{2}, \quad A\bar{C} = (G_4 - \mathrm{i}G_5)/\sqrt{2}, \\ B\bar{C} &= (G_6 - \mathrm{i}G_7)/\sqrt{2}, \quad B\bar{A} = (G_1 + \mathrm{i}G_2)/\sqrt{2}, \\ C\bar{A} &= (G_4 + \mathrm{i}G_5)/\sqrt{2}, \quad C\bar{B} = (G_6 + \mathrm{i}G_7)/\sqrt{2}. \quad (3) \end{aligned}$$

The isospin singlet is symmetric:

$$G_0 = (A\bar{A} + B\bar{B} + C\bar{C})/\sqrt{3}. \quad (4)$$

It will be later realized that G_0 is a weak gauge boson with isospin zero, $G_1 \sim G_3$ are weak gauge bosons with isospin one, and $G_4 \sim G_8$ are weak gauge bosons with isospin two.

The conserved quantity Q as a total electric charge is quantized in terms of the Gell-Mann–Nishijima formula [11] as the subgroup $U(1)_e$ of the $SU(3)_I$ gauge symmetry via the $SU(2)_\mathrm{L} \times U(1)_Y$ gauge group. The corresponding charge operator $\hat{Q}$ is defined by

$$\hat{Q} = \hat{I}_3 + \hat{Y}/2, \quad (5)$$

where $\hat{I}_3$ is the third component of the isospin operator $\hat{I}$ and $\hat{Y} = \hat{B} - \hat{L}$ is the hypercharge operator $\hat{Y}$ with the baryon number operator $\hat{B}$ and the lepton number operator $\hat{L}$.

2.2 Θ vacuum: Weak CP asymmetry

The $(V+A)$ current anomaly is taken into account in analogy with the axial current anomaly [12], which is linked to the Θ vacuum in QCD as a gauge theory [13,6]. The normal vacuum is unstable and the tunnelling mechanism is possible between all possible vacua. The true vacuum must be a superposition of the various vacua, each belonging to some different homotopy class. The effective $\bar{\Theta}$ term in QCD involves both the bare Θ term relevant for pure QCD vacuum and the phase of the quark mass matrix relevant for electroweak effects. Only the latter is here considered as the Θ vacuum responsible for the fermion mass and is added as a single, additional non-perturbative term to the QWD Lagrangian density

$$\mathcal{L}_{\mathrm{QWD}} = \mathcal{L}_\mathrm{P} + \Theta\frac{g_i^2}{16\pi^2}\mathrm{Tr}F^{\mu\nu}\tilde{F}_{\mu\nu}, \quad (6)$$

where $\mathcal{L}_\mathrm{P}$ is the perturbative Lagrangian density (1), $F^{\mu\nu}$ is the field strength tensor, and $\tilde{F}_{\mu\nu}$ is the dual of the field strength tensor. Since the $F\tilde{F}$ term is a total derivative, it does not affect the perturbative aspects of the theory. Such a term in the QWD Lagrangian represents the $(V + A)$ current anomaly, violates CP, T, and P symmetries, and causes the DSSB of local gauge symmetry and global chiral symmetry. Since the Lagrangian density for weak interactions (1) is completely symmetric under the $SU(3)_I$ gauge transformation, the conservation of isospin degrees of freedom holds. Spatially longitudinal components of the gauge fields parameterized by scalar fields condense and then subtract the vacuum energy in the broken phase as the system expands; scalar fields replace the roles of Higgs bosons in the electroweak interactions. Details are given in the following subsections.

2.3 Dynamical spontaneous symmetry breaking

DSSB requires the Θ term and scalar fields. Scalar fields with spin zero, which replace the roles of Higgs particles, are postulated as spatially longitudinal components of gauge fields. Therefore, scalar fields always have the same symmetry as the gauge symmetry of the gauge fields. DSSB is triggered by the Θ term in (6) as the surface term, which demands the boundary condition breaking the discrete symmetries. Due to the Θ term, some of the scalar fields condense. There are two types of transverse gauge fields which are left-handed (or vector) gauge fields and right-handed (or axial-vector) gauge fields. There are also two types of scalar fields which are real scalar fields and pseudo-scalar fields. Among the scalar fields, pseudo-scalar fields condense while real scalar fields remain. During this process, continuous and discrete symmetries are simultaneously broken. This mechanism makes the gauge bosons massive and massive gauge bosons break the discrete symmetries in the fermion spectra. Masses of fermions and bosons are thus acquired as a consequence of DSSB. In fact, gauge bosons become less massive and fermions become more massive as the condensation increases. In the phase transition from the $SU(3)_I$ to the $SU(2)_\mathrm{L} \times U(1)_Y$ gauge symmetry, more massive gauge bosons with isospin 2 become less important and less massive gauge bosons with isospin 1 become more important. The scale of the gauge boson mass is related to the scale of the vacuum energy. Left-handed and right-handed fermions are classified through the weak phase transition. Overall, these processes simultaneously and dynamically take place and both continuous and discrete symmetries are spontaneously broken. In summary, the DSSB mechanism involves the mass generation of fermions and bosons, discrete symmetry breaking in fermions and bosons, and continuous symmetry breaking in scalar and gauge fields.

A fermion possesses three intrinsic isospin degrees of freedom; in group theoretical language, $3 \otimes 3 \otimes 3 = 10 \oplus 8 \oplus 8 \oplus 1$. Two octets are mixed to form leptons and quarks, which is discussed in the following section. In this scheme, the masses of the gauge bosons are reduced due to the condensation of the scalar fields, which are postulated as spatially longitudinal components of gauge bosons. This is very analogous to the mass generation of the electroweak interactions through the Higgs mechanism [8], but it is different in that the gauge boson mass decreases as the condensation increases as shown in the following subsection. Gauge fields are generally decomposed by charge non-singlet–singlet symmetries on the one hand and by even–odd discrete symmetries on the other hand: they have dual properties in charge and discrete symmetries. The Higgs particles are not necessary in this case, since spatially longitudinal components of the gauge bosons play the same role as the Higgs particles. This scheme introduces dynamical symmetry breaking without any free parameters except the weak coupling constant. The $SU(3)_I$ gauge symmetry is spontaneously broken to the $SU(2)_{\mathrm{L}} \times U(1)_Y$ gauge symmetry by the condensation of scalar fields, postulated as spatially longitudinal components of gauge fields. DSSB consists of two simultaneous mechanisms; the first mechanism is the explicit symmetry breaking of the gauge symmetry, which is represented by the isospin factor i_f and the weak coupling constant g_i, and the second mechanism is the spontaneous symmetry breaking of the gauge fields, which is represented by the condensation of pseudo-scalar fields.

Effective gauge boson interactions can, from the Lagrangian density (6), be written

$$\triangle\mathcal{L}^{\mathrm{e}} = -\frac{1}{2}\mathrm{Tr}F_{\mu\nu}F^{\mu\nu} + \Theta\frac{g_i^2}{16\pi^2}\mathrm{Tr}F^{\mu\nu}\tilde{F}_{\mu\nu}. \qquad (7)$$

Spatially longitudinal components of the gauge bosons are postulated as the $SU(3)$ symmetric scalar fields. Four scalar field interactions are parameterized by the typical potential

$$V_{\mathrm{e}}(\phi) = V_0 + \mu^2\phi^2 + \lambda\phi^4, \qquad (8)$$

where $\mu^2 < 0$ and $\lambda > 0$ are demanded for spontaneous symmetry breaking. The first term of the right-hand side corresponds to the bare vacuum energy density representing the zero-point energy. The vacuum field ϕ is shifted by an invariant quantity $\langle\phi\rangle$, which satisfies

$$\langle\phi\rangle^2 = \phi_0^2 + \phi_1^2 + \cdots + \phi_i^2, \qquad (9)$$

where the condensation of pseudo-scalar fields is $\langle\phi\rangle = (-\mu^2/(2\lambda))^{1/2}$. DSSB is relevant for the surface term in (7), $\Theta(g_i^2/16\pi^2)\mathrm{Tr}F^{\mu\nu}\tilde{F}_{\mu\nu}$, which explicitly breaks down the $SU(3)_I$ gauge symmetry to the $SU(2)_{\mathrm{L}} \times U(1)_Y$ gauge symmetry through the condensation of pseudo-scalar bosons; scalar fields are also broken from the $SU(3)$ to the $SU(2) \times U(1)$ symmetry in this case. Θ can be assigned to a dynamic parameter by

$$\Theta = 10^{-61}\rho_G/\rho_{\mathrm{m}}, \qquad (10)$$

with the matter energy density ρ_{m} and the vacuum energy density $\rho_G = M_G^4$. The details of the Θ constant will be discussed in Sect. 6. As the system expands, pseudo-scalar bosons condense and accordingly the gauge boson masses are reduced.

For QWD, being a $SU(3)_I$ gauge theory, there are nine weak gauge bosons ($n_i^2 = 3^2 = 9$), which consist of one singlet gauge boson G_0 with $i = 0$, three degenerate gauge bosons $G_1 \sim G_3$ with $i = 1$, and five degenerate gauge bosons $G_4 \sim G_8$ with $i = 2$ as shown in (2). In the case of isospin 1 gauge bosons, G_3 has the third component 0 and G_1 and G_2 have the third component 1. In the case of isospin 2 gauge bosons, G_8 has the third component 0, G_6 and G_7 have the third component 1, and G_4 and G_5 have the third component 2. For the GWS model, with the $SU(2)_{\mathrm{L}} \times U(1)_Y$ gauge theory, one singlet gauge boson G_0 with $i = 0$, three gauge bosons $G_1 \sim G_3$ with $i = 1$, and one gauge boson G_8 with $i = 2$ are required. This implies that the mixing of G_3 with $i = 1$ and G_8 with $i = 2$ is the same as mixing between the third component 0 gauge bosons, which is represented by the Weinberg mixing angle $\sin^2\theta_{\mathrm{W}} \simeq 1/4$. In the DSSB mechanism of the $SU(3)_I$ to the $SU(2)_{\mathrm{L}} \times U(1)_Y$ gauge symmetry, more massive gauge bosons with $i = 2$ reduce their roles as intermediate vector bosons. Even though their contributions are very weak, they appear in quark flavor mixing.

The concept of $SU(2)$ isospin degree of freedom introduces intrinsic quantum numbers. This means that the intrinsic isospin degree of freedom can be regarded as an intrinsic angular momentum such as the spin degree of freedom. The isospin principal number n_i in intrinsic space quantization is very much analogous to the principal number n in extrinsic space quantization and the intrinsic angular momenta are analogous to the extrinsic angular momentum. The total angular momentum has the form $\mathbf{J} = \mathbf{L} + \mathbf{S} + \mathbf{I}$, which is the extension of the conventional total angular momentum $\mathbf{J} = \mathbf{L} + \mathbf{S}$. The intrinsic principal number n_i denotes the intrinsic spatial dimension or radial quantization: $n_i = 3$ represents weak interactions as an $SU(3)_I$ gauge theory.

2.4 Effective coupling constant and gauge boson mass

Weak interactions are generated by the emission and absorption of weak gauge bosons. Gauge bosons are the analogs of photons for the electromagnetic force and gluons for the color force. Contrary to the photon, the gauge boson must be massive, otherwise it would have been directly produced in weak decays. G_{F} is replaced by gauge boson propagation $2^{1/2}i_f g_i^2/8(k^2 - M_G^2)$ with the gauge boson mass M_G and, in contrast to the dimensionless coupling constant g_i and the isospin factor i_f, has the dimension of inverse energy squared.

The weak interaction amplitude is thus of the form

$$\mathcal{M} = -\frac{i_f g_i^2}{4} J^\mu \frac{1}{k^2 - M_G^2} J_\mu^\dagger = \sqrt{2} G_F J^\mu J_\mu^\dagger, \quad (11)$$

where $\mathcal{M}$ is the product of two universal current densities and i_f is the isospin factor, which is defined by

$$i_f = \frac{1}{4}(i_3^\dagger \lambda^a i_1)(i_2^\dagger \lambda_a i_4), \quad (12)$$

with the isospin fields, i_i with $i = 1 \sim 4$, in analogy with the color factor c_f in QCD. The $(V - A)$ current is conserved but the $(V + A)$ current is not conserved. If $k^2 << M_G^2$, the effective coupling constant becomes

$$\frac{G_\mathrm{F}}{\sqrt{2}} = -\frac{i_f g_i^2}{8(k^2 - M_G^2)} \simeq \frac{g_w^2}{8M_G^2}, \quad (13)$$

with $g_w = i_f g_i$, and the weak currents interact essentially at a point. That is, in the low momentum transfer, the propagation between the currents disappears. The above equation prompts the idea that weak interactions are weak not because g_i is much smaller than e but because M_G^2 is large. Indeed, the two coupling constants are related by $e^2 = i_f g_i^2 = g_i^2/16$ and around the energies of the intermediate vector bosons, weak interactions would become of a strength comparable to the electroweak interactions.

The Lagrangian density for an $SU(2)_\mathrm{L} \times U(1)_Y$ gauge theory has the same form as the one for QWD as an $SU(3)_I$ gauge theory:

$$\mathcal{L}_\mathrm{GWS} = -\frac{1}{2}\mathrm{Tr} G_{\mu\nu} G^{\mu\nu} + \sum_{i=1} \bar{\psi}_i \mathrm{i}\gamma^\mu D_\mu \psi_i + \Theta \frac{g_w^2}{16\pi^2} \mathrm{Tr} G^{\mu\nu} \tilde{G}_{\mu\nu}, \quad (14)$$

where the bare Θ term [13] is a non-perturbative term added to the perturbative Lagrangian density with $SU(2)_\mathrm{L} \times U(1)_Y$ gauge invariance. The above Lagrangian density has the same form as the GWS model in the fermion and gauge boson parts but the Θ term replaced with the Higgs term in the GWS model. The Θ term is apparently odd under the P, T, C, and CP operations. The coupling constant $g_w^2 = i_f g_i^2 = \sin^2\theta_\mathrm{W} g_i^2 = g_i^2/4$ is given in terms of the weak coupling constant g_i and the isospin factor i_f.

Since the covariant derivative is changed from $D_\mu = \partial_\mu + \mathrm{i}g_\mathrm{i} G_{\mu a}\lambda^a/2$ in the $SU(3)_I$ gauge theory to $D_\mu = \partial_\mu + \mathrm{i}g_w W_{\mu a}\lambda^a/2 + \mathrm{i}g_y B_\mu$ in the above Lagrangian density, the gauge boson mass term is obtained via

$$\begin{aligned}\triangle\mathcal{L} &= \frac{1}{2}(D_\mu\phi)^2 - \frac{1}{2}g_w^2\langle\phi\rangle^2 W_\mu W^\mu \\ &= \frac{g_w^2}{2}(W_\mu\phi)^2 - \frac{1}{2}g_w^2\langle\phi\rangle^2 W_\mu W^\mu \cdots, \quad (15)\end{aligned}$$

where the intermediate vector bosons W_μ and B_μ are defined in the following section and $\langle\phi\rangle$ is the condensation of the pseudo-scalar boson. Note that the vacuum energy due to the scalar boson ϕ is shifted with respect to its condensation $\langle\phi\rangle$; this implies that the condensation subtracts the zero-point energy in the system. The coupling constant $i_f g_i^2$ and the vacuum expectation value $\langle\phi\rangle$ for the condensation of the pseudo-scalar field make the gauge boson at low energy less massive: the gauge boson mass is generally defined by

$$M_G^2 = M_H^2 - i_f g_i^2 \langle\phi\rangle^2 = i_f g_i^2[\phi^2 - \langle\phi\rangle^2], \quad (16)$$

where $M_H = i_f^{1/2} g_i \phi$ indicates the unification gauge boson mass at a phase transition just above the weak phase transition, ϕ denotes the real scalar boson and $\langle\phi\rangle$ stands for the condensation of the pseudo-scalar boson. Note that the isospin factor i_f used in (16) is the symmetric factor for a gauge boson with even parity and the asymmetric factor for a gauge boson with odd parity. This process leads to the breaking of the discrete symmetries P, C, T, and CP, as is discussed in the following subsection. The crucial point is that the more massive gauge boson at higher energies becomes lighter through its condensation at low energies. The Fermi weak interaction constant at low momentum transfer can be, by the expression (13), related to the gauge boson mass. The mass must be identical to the inverse of the screening length, that is, $M_G = 1/l_\mathrm{EW} \simeq G_\mathrm{F}^{1/2}$. The gauge boson mass M_G is related to the effective vacuum energy density $V_\mathrm{e}(\phi)$ in (8) by $V_\mathrm{e} = M_G^4$: $V_0 = M_H^4 \approx 10^{12}\,\mathrm{GeV}^4$, $\mu^2 = -2i_f g_i^2 M_H^2 \approx -3 \times 10^6\,\mathrm{GeV}^2$, $\lambda = i_f^2 g_i^4 \approx 0.01$ for $M_G \approx 10^2\,\mathrm{GeV}$, $M_H \approx 10^3\,\mathrm{GeV}$, and $\alpha_i \approx 0.12$. The weak vacuum represented by massive gauge bosons is quantized by the maximum wavevector mode $N_R \approx 10^{30}$, the total gauge boson number $N_G = 4\pi N_R^3/3 \approx 10^{91}$, and the gauge boson number density $n_G = \Lambda_\mathrm{EW}^3 \approx 10^{47}\mathrm{cm}^{-3}$. The Yukawa potential associated with the massive gauge boson is given by

$$V(r) = \sqrt{\frac{i_f g_i^2}{4\pi}} \frac{\mathrm{e}^{-M_G(r - l_\mathrm{EW})}}{r}, \quad (17)$$

which shows the short range interaction.

2.5 Breaking of discrete symmetries

The normal vacuum proceeds to the physical vacuum through the condensation mechanism represented by DSSB. DSSB by the condensation of scalar fields, which are postulated as spatially longitudinal components of the gauge bosons, leads to the breaking of unitarity symmetry as well as the breaking of discrete symmetries. Electromagnetic and weak interactions preserve different discrete symmetries. The electromagnetic interaction violates isospin symmetry but preserves the C, P, and T symmetries. The weak interaction separately violates all of them but is invariant under the combined CPT symmetry.

In the above, it is briefly mentioned that the $SU(2)_\mathrm{L} \times U(1)_Y$ gauge theory for electroweak interactions has its origin in the $SU(3)_I$ gauge theory for weak interactions as shown by the triggering of DSSB by the condensation of the scalar bosons. The condensation is also the origin of the discrete symmetry violation which is observed in electroweak interactions. The discrete symmetries P, C, and T are broken down explicitly by the condensation of the scalar bosons; the product symmetry CPT remains intact and CP conserves approximately. Since the condensation of scalar boson is relevant for the vector and axial-vector currents, the $(V-A)$ doublet current is conserved in the $SU(2)_\mathrm{L}$ weak theory, $\partial_\mu J_\mu^{1-\gamma^5}=0$, but the $(V+A)$ singlet current is not conserved,

$$\partial_\mu J_\mu^{1+\gamma^5} = \frac{N_f g_i^2}{16\pi^2}\mathrm{Tr}F^{\mu\nu}\tilde{F}_{\mu\nu}, \tag{18}$$

with the flavor number N_f. The reason for the parity non-conservation is the scalar boson condensation or the fermion condensation represented by the condensation number N_{sc} in the fermion mass generation as discussed later. The $SU(3)_I$ symmetry is broken by DSSB, in which the right-handed symmetry is not manifest in the particle spectrum such as the absence of right-handed neutrinos explained by P violation [14]. The Cooper pairing between matter particles violates C symmetry and the Θ vacuum violates T symmetry explicitly. The inclusion of the third quark generation is represented by the Kobayashi–Maskawa (KM) matrix [15] which contains P violation and CP symmetry violation. The assumption of the KM matrix has a verifiable consequence for the decay of the K mesons. CP violation observed in the neutral kaon decay [16] with the probable value $\Theta \simeq 10^{-3}$ indicates T violation because CPT symmetry is conserved. The electric dipole moment of electrons is observed to be $d_e = -1.5\times 10^{-26}\, e\mathrm{cm}$ [17], which implies $\Theta \simeq 10^{-4}$ if the effective dipole length of electrons $l_e \simeq G_\mathrm{F} m_e \simeq 10^{-22}\,\mathrm{cm}$ is used. A possible lepton–antilepton asymmetry is suggested as a consequence of C, T, and CP violation during the DSSB of the $SU(3)_I$ symmetry to the $SU(2)_\mathrm{L}\times U(1)_Y$ symmetry: the electron asymmetry $\delta_e \simeq 10^{-7}$ [18,19] is suggested as a consequence of the $U(1)_Y$ gauge theory. The lepton number is not conserved above the weak scale, but the lepton number is conserved below the weak scale as illustrated by the $U(1)_Y$ gauge theory in the weak interactions.

The DSSB of gauge symmetry and chiral symmetry induces the $(V+A)$ current anomaly represented by the value $\Theta \approx 10^{-4}$ at the weak scale. This implies the reduction of zero modes through the scalar boson condensation. The Θ vacuum as the physical vacuum is achieved from the normal vacuum, which possesses a larger symmetry group than the physical vacuum. The instanton mechanism, vacuum tunnelling, is expected in the Euclidean spacetime. The Θ vacuum term represents the surface term since it is a total derivative and decreases as the system expands. The condensation of vacuum decreases the mass of gauge boson, which causes the expansion of the system during a phase transition: this is the source of the exponential inflation as expected by the Higgs mechanism.

Photons as NG bosons in DSSB play the role of massless gauge bosons responsible for the $U(1)_e$ gauge theory. Photons are generated as massless gauge bosons from the weak interactions by DSSB whereas gauge bosons as intermediate vector bosons are generated as massive gauge bosons by DSSB. Photons mediate the Coulomb interaction in the static limit as a result of the symmetry breaking of the $SU(2)_\mathrm{L}\times U(1)_Y$ to $U(1)_e$ gauge symmetry. Details of photon dynamics generation are described in the following section.

3 Generation of electroweak interactions

In the previous section, the common features in DSSB are addressed, and in this section the precise generation of the GWS model from QWD is more in particular focused on and a resolution for problems of the GWS model is suggested.

The generation of the GWS model by $SU(2)_\mathrm{L}\times U(1)_Y \to U(1)_e$ electroweak interactions [1] from QWD as an $SU(3)_I$ gauge theory will now be considered. Spatially longitudinal components of the gauge bosons parameterized by scalar fields play the role of Higgs bosons in the electroweak interactions, so that the GWS model, $SU(2)_\mathrm{L}\times U(1)_Y$ gauge theory, dynamically results from QWD being an $SU(3)_I$ gauge theory. QWD, an $SU(3)_I$ gauge theory, generates the electroweak theory, an $SU(2)_\mathrm{L}\times U(1)_Y$ gauge theory, through the condensation of the scalar field. The condensation of the scalar boson also produces DSSB yielding the $U(1)_e$ gauge symmetry as expected by the Higgs mechanism of the electroweak interactions. The covariant derivative $D_\mu = \partial_\mu + \mathrm{i}g_\mathrm{i} G_{\mu a}\lambda^a/2$ of the $SU(3)_I$ gauge theory becomes $D_\mu = \partial_\mu + \mathrm{i}g_w W_{\mu a}\lambda^a/2 + \mathrm{i}g_y B_\mu$ of the $SU(2)_\mathrm{L}\times U(1)_Y$ gauge theory. The weak coupling constant g_i is the unification of the electroweak charge g_w and the hypercharge g_y at the weak scale.

There are several supporting clues for the generation of the electroweak interactions being an $SU(2)_\mathrm{L}\times U(1)_Y$ gauge theory from the weak interactions as an $SU(3)_I$ gauge theory: parity violation, the Fermi weak coupling constant, the Weinberg angle, three generations as doublets of leptons and quarks, the Cabbibo angle, and the weak coupling constant g_w. Since scalar field generators do commute with some octet gauge bosons, massive gauge bosons reduce their masses through the condensation of pseudo-scalar bosons. This is exactly as expected from electroweak interactions of the $SU(2)_\mathrm{L}\times U(1)_Y$ gauge symmetry since three intermediate vector bosons, $W^\pm$ and Z (or W^3), associated with the generators $\lambda_1 \sim \lambda_3$, are massive and the photon is massless. Through the condensation of pseudo-scalar bosons, DSSB from $SU(2)_\mathrm{L}\times U(1)_Y$ to $U(1)_e$ is accomplished. The weak gauge bosons $G_4 \sim G_7$ with isospin two are heavier than the weak gauge bosons $G_1 \sim G_3$ with isospin one and their contribution almost disappears during the phase transition of $SU(3)_I$ to $SU(2)_\mathrm{L}\times U(1)_Y$ symmetry: the detailed concept of isospin will be discussed on treating intrinsic quantum numbers. During DSSB, parity violation and charge conjugation violation are maximal due to the condensation of pseudo-scalar fields. The linear combination of the diagonal isospin octet generators in two octets of triplet isospin combinations generates the three families of an isospin doublet in leptons and quarks. The mass generation scheme for fermions is suggested in terms of the DSSB of chiral symmetry without introducing new input parameters, unlike the GWS model. In this scheme, the $SU(2)_{\rm L}\times U(1)_Y$ symmetry and the $U(1)_e$ symmetry using the symmetric isospin factors, $i_f^{\rm s}=(i_f^z,i_f^w,i_f^y,c_f^e)=(1/3,1/4,1/12,1/16)$, are applied to the typical electroweak interactions.

The conclusive clues are presented in the following: the Fermi weak coupling constant, the Weinberg angle and neutral currents, the three generations of leptons and quarks, the electroweak coupling constants, and the electroweak flavor mixing angle for quarks.

3.1 Fermi weak coupling constant

As described by the previous section, the electroweak interaction amplitude takes the form of

$$\mathcal{M}=-\frac{g_w^2}{2}J^\mu\frac{1}{k^2-M_W^2}J_\mu^\dagger=\frac{4}{\sqrt{2}}G_{\rm F}J^\mu J_\mu^\dagger, \tag{19}$$

where the Fermi weak coupling constant

$$\frac{G_{\rm F}}{\sqrt{2}}=-\frac{g_w^2}{8(k^2-M_W^2)}\simeq\frac{g_w^2}{8M_W^2} \tag{20}$$

has the dimension of inverse energy squared. The isospin factor $i_f=\frac{1}{4}(i_3^\dagger\lambda^a i_1)(i_2^\dagger\lambda_a i_4)$ is defined in terms of the isospin field i. Note that (20) is exactly identical to (13), since $g_z=i_f g_i=g_i/3$, $g_w=g_z\cos\theta_{\rm W}$, $M_G^2=M_W^2/i_f$, and $M_Z=M_W/\cos\theta_{\rm W}$: the isospin factor $i_f=1/3$ for the symmetric sextet configuration is discussed in the following subsections. The gauge boson mass M_G is connected with the electroweak cutoff scale $\Lambda_{\rm EW}$.

3.2 Weinberg angle and neutral current

Another clue for the generation of the $SU(2)_{\rm L}\times U(1)_Y$ to the $SU(3)_I$ symmetry comes from the Weinberg angle $\theta_{\rm W}$ which represents the mixing of G_3 and G_8. The experimental value $\sin^2\theta_{\rm W}\approx 0.234$ [20] is very close to the theoretical value $\sin^2\theta_{\rm W}=g_y^2/(g_w^2+g_y^2)=0.25$ if the coupling constant g_w for the $SU(2)_{\rm L}$ group and the coupling constant g_y for the $U(1)_Y$ group are determined from the $SU(3)_I$ group; the theoretical value of $\tan\theta_{\rm W}=g_y/g_w$ is just the ratio, $1/3^{1/2}$, between the gauge fields G_3 and G_8 from the $SU(3)_I$ gauge symmetry. The neutral weak coupling constant $g_z=(g_w^2+g_y^2)^{1/2}$ is defined and accordingly $\sin\theta_{\rm W}=g_y/g_z$ and $\cos\theta_{\rm W}=g_w/g_z$; g_w and g_y are related by a number known as the isospin factor of the coupling constants. Notice that the experimental value 0.236 is much closer to the theoretical prediction $\sin^2\theta_{\rm W}=0.25$ of this scheme than $\sin^2\theta_{\rm W}=3/8$ of the $SU(5)$ gauge theory [3] at the tree level.

The Weinberg angle is closely related to massive gauge boson and massless photon generation. The gauge mass terms come from (16), evaluated at the shifted vacuum $\phi'^2=\phi^2-\langle\phi\rangle^2$ with scalar boson ϕ and condensed scalar boson $\langle\phi\rangle$. The relevant terms after the phase transition of the $SU(3)_I$ symmetry to the $SU(2)_{\rm L}\times U(1)_Y$ symmetry are

$$\begin{aligned}&\phi'^2(g_{\rm i}G_\mu^a\lambda^a)(g_{\rm i}G^{\mu b}\lambda^b)\\&\quad\to\phi'^2[g_w^2(G_\mu^1)^2+g_w^2(G_\mu^2)^2+(-g_wG_\mu^3+g_yG_\mu^8)^2].\end{aligned} \tag{21}$$

Recall that $G_\mu^1\sim G_\mu^3$ and G_μ^8 are respectively equivalent to $W_\mu^1\sim W_\mu^3$ and B_μ in the GWS electroweak model. There are massive bosons

$$W_\mu^\pm=\frac{1}{\sqrt{2}}(G_\mu^1\mp{\rm i}G_\mu^2), \tag{22}$$

$$Z_\mu^0=\cos\theta_{\rm W}G_\mu^3-\sin\theta_{\rm W}G_\mu^8, \tag{23}$$

where their masses are

$$\begin{aligned}M_W^2&=g_w^2(\phi^2-\langle\phi\rangle^2),\\M_Z^2&=M_W^2/\cos^2\theta_{\rm W}=i_fg_i^2[\phi^2-\langle\phi\rangle^2]\\&=\frac{1}{3}g_i^2[\phi^2-\langle\phi\rangle^2],\end{aligned}$$

with $M_H=i_fg_i^2\phi^2$ respectively. The fourth vector identified as the photon, orthogonal to Z^0, remains massless:

$$A_\mu=\sin\theta_{\rm W}G_\mu^3+\cos\theta_{\rm W}G_\mu^8, \tag{24}$$

with the mass $M_A=0$. Photons play the role of massless gauge bosons responsible for the $U(1)_e$ gauge theory. They are understood as NG bosons during DSSB [9]. Two physical neutral gauge fields Z_μ and A_μ are orthogonal combinations of the gauge fields G_μ^3 and G_μ^8 with the mixing angle $\theta_{\rm W}$. The mixing represents the mixing of gauge bosons with the 0 third component of isospin, G_μ^3 with isospin 1 and G_μ^8 with isospin 2. The electromagnetic current is the combination of the two neutral currents J_μ^3 and j_μ^y. The generators of this scheme satisfy $\hat{Q}=\hat{I}_3+\hat{Y}/2$ as shown in (5) so that

$$j_\mu^e=J_\mu^3+j_\mu^y/2. \tag{25}$$

The hypercharge operator $\hat{Y}=\hat{B}-\hat{L}$ is used and applied in the subsection of the three generations of leptons and quarks. The interaction in the neutral current sector can be given by

$$\begin{aligned}&-{\rm i}g_wJ_\mu^3G^{3\mu}-{\rm i}g_yj_\mu^yG^{8\mu}/2\\&\quad=-{\rm i}ej_\mu^eA^\mu-{\rm i}g_z[J_\mu^3-\sin^2\theta_{\rm W}j_\mu^e]Z^\mu,\end{aligned} \tag{26}$$

where the relation

$$e=g_w\sin\theta_{\rm W}=g_y\cos\theta_{\rm W}=g_z\cos\theta_{\rm W}\sin\theta_{\rm W} \tag{27}$$

Table 1. Weak isospin and hypercharge quantum numbers of leptons and quarks

Leptons	I	I_3	Y	Q
ν_e	1/2	1/2	−1	0
$e_{\rm L}$	1/2	−1/2	−1	−1
$e_{\rm R}$	0	0	−2	−1

Quarks	I	I_3	Y	Q
$u_{\rm L}$	1/2	1/2	1/3	2/3
$d_{\rm L}$	1/2	−1/2	1/3	−1/3
$u_{\rm R}$	0	0	4/3	2/3
$d_{\rm R}$	0	0	−2/3	−1/3

is used. The Weinberg angle in the neutral current interaction of (26) is further addressed by

$$\begin{aligned} &-{\rm i}\frac{g_w}{\cos\theta_{\rm W}} j_\mu^{NC} Z^\mu \\ &= -{\rm i} g_z \bar\psi\gamma^\mu\left[\frac{1}{2}(1-\gamma^5)I_3 - \sin^2\theta_{\rm W} Q\right]\psi Z_\mu \\ &= -{\rm i} g_z \bar\psi\gamma^\mu \frac{1}{2}(g_V - g_A\gamma^5)\psi Z_\mu, \end{aligned} \quad (28)$$

where j_μ^e is the electric current density. The vector and axial-vector couplings, g_V and g_A, are determined by

$$g_V = I_3 - 2\sin^2\theta_{\rm W} Q, \quad g_A = I_3. \quad (29)$$

Experimental values for the vector and axial-vector coupling from neutrino-electron data [21] are reasonably in good agreement with the isospin quantum numbers shown in Table 1, which is obtained by the GWS model, when the Weinberg angle is $\sin\theta_{\rm W} = 1/2$; for the electron, the experimental values are $g_A = -0.52$ and $g_V = 0.06$, and the theoretical values are $g_A = -0.5$ and $g_V = 0$.

3.3 Electroweak coupling constants

During the phase transition from the $SU(3)_I$ gauge theory to the $SU(2)_{\rm L} \times U(1)_Y$ gauge theory, two fermion interactions with triplet isospins are represented by $3\otimes 3 = \bar{3}\oplus 6$ in group theoretical language. Similarly to the strong interactions, the triplet isospin charges (A, B, C) are introduced. For two fermions, a triplet with asymmetric combinations becomes

$$(AB - BA)/\sqrt{2}, \quad (BC - CB)/\sqrt{2}, \quad (CA - AC)/\sqrt{2}, \quad (30)$$

and a sextet of symmetric combinations becomes

$$\begin{gathered} AA, \quad BB, \quad CC, \\ (AB + BA)/\sqrt{2}, \ (BC + CB)/\sqrt{2}, \quad (CA + AC)/\sqrt{2}. \end{gathered} \quad (31)$$

The isospin factor $i_f = -2/3$ for the triplet configuration is obtained for the fermion–fermion interactions. Since six pairs with the same isospin factor are, together with the normalization factor 1/6, taken into account, the isospin factor is $i_f = 1/3$. The completely symmetric sextet configuration is related to the three generations of leptons and quarks; sextet members might lead to three generations of the $SU(2)_{\rm L}$ isospin symmetry after the electromagnetic phase transition: $SU(2)_{\rm L} \times U(1)_Y \to U(1)_e$. The involved gauge bosons in the $SU(2)_{\rm L} \times U(1)_Y$ gauge theory are $G_1 \sim G_3$ and G_8. Note that the coupling constant for the charge neutral current is $\alpha_z = i_f\alpha_i = \alpha_i/3$. Leptons are asymmetric in spin and isospin states while quarks are symmetric in spin and isospin states.

This scheme explains the weak coupling constant consistent with experimental values through the data of muon decay [5]. The observed muon lifetime and mass give a Fermi weak coupling

$$G_{\rm F} = \frac{\sqrt{2}}{8}\left(\frac{g_w}{M_W}\right)^2 = 1.166 \times 10^{-5}\,{\rm GeV}^{-2}. \quad (32)$$

The corresponding value of the weak coupling constant g_w is given by $g_w \approx 0.61$ and the weak fine structure constant becomes

$$\alpha_{\rm w} = g_w^2/4\pi \approx 0.03. \quad (33)$$

On the other hand, since $\alpha_e = 1/137$, $\alpha_e = \alpha_{\rm w}\sin^2\theta_{\rm W}$, $\alpha_z\cos^2\theta_{\rm W} = \alpha_{\rm w}$, $\alpha_z = i_f\alpha_i$, and $\sin^2\theta_{\rm W} = 1/4$, the prediction of $\alpha_z = g_z^2/4\pi = 0.04$ and $\alpha_i = g_i^2/4\pi \simeq 0.12$ at the weak scale is obtained. In summary, the coupling constant hierarchy in the weak interactions is α_i, $\alpha_z = \alpha_i/3 \simeq 0.04$, $\alpha_{\rm w} = \alpha_i/4 \simeq 0.03$, $\alpha_y = \alpha_i/12 \simeq 0.01$, and $\alpha_e = \alpha_i/16 \simeq 1/133$ at the weak scale. The isospin factor $i_f = 1/4$ for $\alpha_{\rm w} = \alpha_i/4$ represents the coupling between parallel $SU(2)$ isospins.

3.4 Three generations of leptons and quarks: Weak isospin and charge quantum number

This scheme exhibits three generations of leptons and quarks holding family symmetry, which is one of the fundamental questions in particle physics. The left-handed lepton doublets are

$$\begin{pmatrix}\nu_e \\ e\end{pmatrix}_{\rm L}, \begin{pmatrix}\nu_\mu \\ \mu\end{pmatrix}_{\rm L}, \begin{pmatrix}\nu_\tau \\ \tau\end{pmatrix}_{\rm L} \quad (34)$$

and the right-handed lepton singlets are $e_{\rm R}, \mu_{\rm R}, \tau_{\rm R}$. The left-handed quark doublets are

$$\begin{pmatrix}u \\ d\end{pmatrix}_{\rm L}, \begin{pmatrix}c \\ s\end{pmatrix}_{\rm L}, \begin{pmatrix}t \\ b\end{pmatrix}_{\rm L} \quad (35)$$

and the right-handed quark singlets are $u_{\rm R}$ or $d_{\rm R}$, $c_{\rm R}$ or $s_{\rm R}$, and $t_{\rm R}$ or $b_{\rm R}$. The detailed scenarios of the isospin and charge quantum numbers are as follows.

The $SU(3)_I$ symmetry represents, in group theoretical language, $3\otimes 3\otimes 3 = 10\oplus 8\oplus 8\oplus 1$ for three fermion combinations with triplet isospins. When triplet isospin charges A, B, C are introduced, the two octets are partially asymmetric under the interchange of isospin. One octet is asymmetric under the interchange of the first and second isospins, (A, B), and the other octet is asymmetric under the interchange of the first and third isospins, (A, C), or the second and third isospins, (B, C). The linear combination of two octets yields the fermion families with symmetric isospin configurations. Three families of leptons are created as the linear combination of diagonal isospin components of three $SU(2)$ subgroups, I, U, and V isospins, in two octets. The phase transition of the $SU(3)_I \to SU(2)_\mathrm{L} \times U(1)_Y \to U(1)_e$ gauge theory provides us with the electric charge quantization $\hat{Q} = \hat{I}_3 + \hat{Y}/2$ with $\hat{Y} = \hat{B} - \hat{L}$. For example, the linear combination of generators λ_3^a and λ_3^s is for the left-handed lepton doublet, and the linear combination of the generators λ_8^a and λ_3^s is for the right-handed lepton singlet, where the superscripts a and s represent the asymmetric and symmetric states under interchanging the first two particles, respectively. The generators are dual in parity:

$$\lambda_3^\mathrm{a}/2 = \mathrm{diag}(1, -1, 0)/2, \quad \lambda_3^\mathrm{s}/2 = \mathrm{diag}(1, 1, 0)/2,$$
$$\lambda_8^\mathrm{a}/2\sqrt{3} = \mathrm{diag}(1, 1, -2)/6, \quad \lambda_8^\mathrm{s}/2\sqrt{3} = \mathrm{diag}(1, 1, 2)/6.$$

Note that the rotation by the Weinberg angle is included in the coefficients. Three generations of quarks are also created as a linear combination of the diagonal isospin components of the I, U and V subgroups: for instance, the linear combination of λ_3^a and λ_8^s for the left-handed quark doublet and the linear combination of λ_8^a and λ_8^s for the right-handed quark singlet. The mass difference among the three families depends on the condensation of the pseudo-scalar bosons. The $SU(3)_I$ gauge symmetry is broken as a result of the condensation of the pseudo-scalar bosons. The λ_3^s is relevant for the lepton families with $I_3 = 0$ and $Y = -L = -1$, the λ_8^s is relevant for the quark families with $I_3 = 0$ and $Y = B = 1/3$, the λ_3^a is relevant for the left-handed doublets $I_3 = \pm 1/2$ and $Y = 0$, and the λ_8^a is relevant for the right-handed singlets with $I_3 = 0$ and $Y = -1$. Note that the states of λ_3^a and λ_8^a determine the distinction between the left- and right-handed particles; they are closely related to the axial-vector charges since they hold odd parity while λ_3^s and λ_8^s are related to the vector charges since they hold even parity. A summary for the generation of leptons and quarks is as follows. The linear combination of λ_3^a and λ_3^s generates the left-handed lepton doublets with $I_3 = \pm 1/2$ and $Y = -L = -1$: $\mathrm{diag}(1, -1, 0)/2 - \mathrm{diag}(1, 1, 0)/2 = \mathrm{diag}(0, -1, 0)$. The linear combination of λ_3^a and λ_8^s generates the left-handed quark doublets with $I_3 = \pm 1/2$ and $Y = B = 1/3$: $\mathrm{diag}(1, -1, 0)/2 + \mathrm{diag}(1, 1, 2)/6 = \mathrm{diag}(2, -1, 1)/3$. The linear combination of λ_8^a and λ_3^s generates the right-handed lepton singlets with $I_3 = 0$ and $Y = -2$: $-\mathrm{diag}(1, 1, -2)/6 - \mathrm{diag}(1, 1, 0)/2 = \mathrm{diag}(-2, -2, 1)/3$. The linear combination of λ_8^a and λ_8^s generates the right-handed quark singlets with $I_3 = 0$ and $Y = 4/3$ or with $I_3 = 0$ and $Y = -2/3$: $-\mathrm{diag}(1, 1, -2)/6 + \mathrm{diag}(1, 1, 2)/6 = \mathrm{diag}(0, 0, 2)/3$ or $-\mathrm{diag}(1, 1, -2)/6 - \mathrm{diag}(1, 1, 2)/6 = \mathrm{diag}(-1, -1, 0)/3$, where the element $-1/3$ denotes the down quark and the element $2/3$ denotes the up quark. Weak isospin and electric charge quantum numbers of leptons and quarks are summarized in Table 1. This scheme illustrates the left–right symmetry before DSSB; it is similar to the composite model [22] and the left–right symmetric model of the weak interactions [23].

In other words, each elementary fermion such as the lepton or quark possesses color and isospin degrees of freedom in addition to spin degrees of freedom. The lepton is postulated as a color singlet state while the quark is postulated as a color triplet state so that it may interact through the color charge exchange. All leptons and quarks consist of a lepton octet and a quark octet, in which each family explicitly possesses three generations of isospin doublets with one electric charge unit difference: this is very much analogous to the celebrated eight-fold way in hadron spectra with quark flavor symmetry.

3.5 Electroweak flavor mixing angle for quarks

Further clues are the mixing angles of the KM matrix [15] including the Cabbibo angle [24]. The mixing between the d and s quarks in the decay of the vector boson W^+ is represented by the Cabbibo angle which indicates mixing among the quark families. The mixing of the d and s quark is observed by comparing $\Delta S = 1$ and $\Delta S = 0$ decays:

$$\frac{\Gamma(K^+ \to \mu^+ + \nu_\mu)}{\Gamma(\pi^+ \to \mu^+ + \nu_\mu)} \sim \sin^2 \theta_\mathrm{C},$$
$$\frac{\Gamma(K^+ \to \pi^0 + e^+ + \nu_\mu)}{\Gamma(\pi^+ \to \pi^0 + e^+ + \nu_\mu)} \sim \sin^2 \theta_\mathrm{C}. \quad (36)$$

The mixing between the quarks is extended to the third generation of quarks by the KM matrix which includes three angles and one CP phase angle [15,25]. These mixing angles and this phase angle show the possibility that quark interactions are generated from both the weak interaction of the $SU(3)_I$ gauge symmetry and the strong interaction of the $SU(3)_C$ gauge symmetry.

Recall that electric charges for the left-handed quarks are obtained by the linear combination of generators λ_3^a and λ_8^s; the electric charge matrix is given by $\mathrm{diag}(2/3, -1/3, 1/3)$. The existence of the third component in QWD, which has a quantum number $Y = 2/3$, which is unpredictable from the GWS model, indicates the $SU(3)_I$ flavor symmetry and is related to the mixing of $W^+ = G^{1+i2}$ and G^{4+i5} gauge fields for positively charged gauge fields; for instance, the total current becomes $j_\mu = \cos\theta_\mathrm{C}\, j_\mu^{1+i2} + \sin\theta_\mathrm{C} j_\mu^{4+i5}$ where the Cabbibo angle $\tan^2\theta_\mathrm{C}$ is determined by the mass squared ratio of the two gauge fields G^{1+i2} and G^{4+i5}: $M_W = M_G^{1+i2}$ and $M_G^{4+i5} \simeq 2M_W$. The Cabbibo angle $\sin\theta_\mathrm{C} = (M_G^{1+i2}/M_G^{4+i5})^2 \simeq 1/4$ is thus predicted and is very close to the experimental value $\sin\theta_\mathrm{C} = 0.231$. This is also supported by $\sin^2\theta_\mathrm{C}$ being connected with the mass ratio $\approx 1/20$ of the d and s quarks [26,27]. The Cabbibo angle thus indicates the mixing between the $SU(2)$ subgroups I and U. This can be considered to be conclusive evidence of the massive gauge bosons G_4 and G_5 with the masses $2M_W$ in addition to the three intermediate vector bosons.

The other two mixing angles in the KM matrix respectively represent the mixing between the $SU(2)$ subgroups I and V and the mixing between the subgroups U and V. Since the other mixing angles are experimentally close to 0, the masses of the gauge bosons G_6 and G_7 must be heavier than those of G_1 and G_2. Note that weak CP violation from the decay of the neutral kaon is relevant for the $\Theta \leq 10^{-3}$, and the absence of right-handed neutrinos is a direct consequence of the non-conservation of the $(V+A)$ current under the breaking of the discrete symmetries.

4 Grand unification of strong and electroweak interactions

In this section, the grand unification of QWD and QCD is clarified in terms of the experimental strong and weak coupling constants. A coupling constant hierarchy for weak interactions is suggested in analogy with the coupling constant hierarchy for the strong interactions.

A grand unified group H contains the $SU(3)_I$ group for the weak interactions and the $SU(3)_C$ group for the strong interactions [2,6,7] as subgroups: $H \supset SU(3)_I \times SU(3)_C$. A certain grand unification group H breaks down to the $SU(3)_I \times SU(3)_C$ group at the grand unification scale around 10^3 GeV, which is much lower than the grand unification scale 10^{15} GeV of the GUT [3] being the grand unification of the standard model $SU(3)_C \times SU(2)_L \times U(1)_Y$. This scheme might provide the resolution to the hierarchy problem and the analogy between the $SU(3)_I$ and $SU(3)_C$ symmetries. In the following, grand unification of the weak and strong interactions and the comparison of effective coupling constants are discussed.

4.1 Grand unification of quantum weakdynamics and quantum chromodynamics

There is conclusive evidence for the proposal that QCD for the strong force is unified with QWD for the weak force at the grand unification scale around 10^3 GeV. The fine structure constant α_s for the strong interactions is measured by numerous experiments [4]:

$$\alpha_s(M_Z) \simeq 0.12, \tag{37}$$

which has been evaluated at the momentum of the Z boson mass $q = M_Z$. The fine structure constant α_w for the $SU(2)_L$ weak interaction is given by the data of muon decay [5]:

$$\alpha_w(M_W) \simeq 0.03 \tag{38}$$

at the momentum of the W boson mass $q = M_W$. The two empirical coupling constants (37) and (38) provide the grand unification scale around 10^3 GeV, where the strong coupling constant is the same as the weak coupling constant: $\alpha_h = \alpha_s = \alpha_i$. For the $SU(3)_I$ symmetry, a symmetric sextet configuration in isospin–isospin interactions has the fine structure constant $\alpha_z = i_f \alpha_i = \alpha_i/3$, which is the coupling constant for the gauge boson Z. Since $\alpha_z = \alpha_w/\cos^2\theta_W$ and $\sin^2\theta_W = 1/4$, the fine structure constant for the W boson is $\alpha_w = \alpha_i/4$; the fine structure constant for QWD, $\alpha_i(M_Z) \simeq 0.12$, is thus obtained. This leads to the fine structure constant for the electromagnetic interactions $\alpha_e = \alpha_w \sin^2\theta_W = \alpha_i/16 = 1/133$. In summary, the fine structure constants are given by $\alpha_h = \alpha_s = \alpha_i \simeq 0.12$, $\alpha_z = \alpha_i/3 \simeq 0.04$, $\alpha_w = \alpha_i/4 \simeq 0.03$, $\alpha_y = \alpha_i/12 \simeq 0.01$, and $\alpha_e = \alpha_i/16 \simeq 1/133$ around the weak scale 10^2 GeV. In addition, the electromagnetic interactions give the fine structure constant $4\alpha_e/9 = \alpha_i/36$ for the up quarks and $\alpha_e/9 = \alpha_i/144$ for the down quarks. This explains the group chains $H \supset SU(3)_I \times SU(3)_C$ and $SU(3)_I \supset SU(2)_L \times U(1)_Y \supset U(1)_e$.

Table 2. Relations between conservation laws and gauge theories

Force	Conservation law	Gauge theory
Electromagnetic	Electron	$U(1)_e$
Weak	Lepton (or $B-L$)	$U(1)_Y$
Weak	$V-A$	$SU(2)_L \times U(1)_Y$
Weak	isotope (isospin)	$SU(3)_I$

After the phase transition of the group H, the groups $SU(3)_I$ and $SU(3)_C$ preserve the analogous properties. The isospin coupling constant is given by g_i and the Weinberg weak mixing angle is given by $\sin^2\theta_W = 1/4$. As the energy scale decreases, the $SU(3)_I$ group for isospin interactions breaks down to the $SU(2)_L \times U(1)_Y$ group for the electroweak interactions at the electroweak scale. The electroweak coupling constants are, in summary, $\alpha_z = i_f^z \alpha_i = \alpha_i/3 \simeq 0.04$, $\alpha_w = i_f^w \alpha_i = \alpha_i/4 \simeq 0.03$, $\alpha_y = i_f^y \alpha_i = \alpha_i/12 \simeq 0.01$, and $\alpha_e = i_f^e \alpha_i = \alpha_i/16 \simeq 1/133$ for the symmetric isospin interactions at the weak scale and $-2\alpha_i/3$, $-\alpha_i/2$, $-\alpha_i/6$, and $-\alpha_i/8$ for the asymmetric isospin interactions: $i_f^w = \sin^2\theta_W$ and $i_f^e = \sin^4\theta_W$. The isospin factors introduced are $i_f^s = (i_f^z, i_f^w, i_f^y, i_f^e) = (c_f^b, c_f^n, c_f^z, c_f^f) = (1/3, 1/4, 1/12, 1/16)$ for symmetric interactions and $i_f^a = (-2/3, -1/2, -1/6, -1/8)$ for asymmetric interactions. The symmetric charge factors reflect an intrinsic even parity with repulsive force while the asymmetric charge factors reflect an intrinsic odd parity with attractive force; this suggests electromagnetic duality. The symmetric charge factors reflect an intrinsic even parity with repulsive force while the asymmetric charge factors reflect an intrinsic odd parity with attractive force. As a consequence of gauge theories, conservation laws are expected. Table 2 shows relations between conservation laws and gauge theories in the weak interactions. However, there is a possibility that not the separate conservation of the baryon number and the lepton number hold, but the combined conservation of $(B-L)$ number conservation in the energies above 10^2 GeV.

4.2 Coupling constants for fundamental forces

The new grand unification energy 10^3 GeV rather than the conventional energy 10^{15} GeV is obtained so that the hierarchy problem seems to be resolved. The unification at the order of a TeV energy is consistent with a recent GUT [29]. Dynamical symmetry breaking is adopted instead of the Higgs mechanism; the condensation of pseudo-scalar bosons reduces the mass of the gauge boson and it becomes the source of the system inflation.

QWD has an asymptotic freedom in the weak coupling constant g_i just as the notable characteristic of QCD is the asymptotic freedom due to anti-screening at short distance as a non-Abelian gauge theory according to the renormalization group study [30]. Massive gauge bosons at the grand unification scale produce massive intermediate vector bosons and massive gluons at weak and strong interaction energies respectively. The effective grand unified coupling becomes

$$\frac{G_H}{\sqrt{2}} = -\frac{g_h^2}{8(k^2 - M_G^2)} \simeq \frac{g_h^2}{8M_G^2} \approx 10^{-4}\,\mathrm{GeV}^{-2}, \qquad (39)$$

where the coupling constant g_h holds for the grand unified gauge group H: the effective coupling constant chain becomes $G_H \supset G_\mathrm{F} \times G_R$ with the effective strong coupling constant $G_R/2^{1/2} = g_s^2/8M_G^2 \approx 10\,\mathrm{GeV}^{-2}$. Note that G_H is close to G_F or it is slightly less than G_F, since $\alpha_h \sim \alpha_i \sim \alpha_\mathrm{s}$ at the energy scale 10^3 GeV. The mass of the gauge boson at the grand unification scale is expressed by $M_G^2 = M_H^2$ at a slightly higher phase transition energy above the weak scale. The gauge boson decreases its mass as the condensation increases when the energy scale decreases. At $T \simeq 10^2$ GeV, the gauge boson number density is $n_G = M_G^3 \simeq 10^6\,\mathrm{GeV}^3 \simeq 10^{47}\,\mathrm{cm}^{-3}$, the vacuum energy density is $V_0 = M_G^4 \simeq 10^8\,\mathrm{GeV}^4 \simeq 10^{25}\,\mathrm{gcm}^{-3}$, and the total gauge boson number is $N_G \simeq 10^{91}$, which is a conserved good quantum number.

Of the four fundamental forces in nature, three forces, the electromagnetic, strong, and weak forces, are unified at the grand unification scale. Strong interactions are limited in range to about 10^{-13} cm and are insignificant even at the scale of the atom 10^{-8} cm, but play an important role in binding the nucleus. Weak interactions with an even shorter scale ($\leq 10^{-15}$ cm) do play an important role in weak decay processes. The strengths of the four forces are roughly in orders of magnitude $10, 10^{-2}, 10^{-5}$, and 10^{-41} for the strong, electromagnetic, weak, and gravitational forces, respectively. The difference in strength is more than a factor of $G_R/G_\mathrm{N} \approx 10^{39}$ for the strong and gravitational interactions. The ratio of the electroweak and gravitational forces is also obtained by $G_\mathrm{F}/G_\mathrm{N} \approx 10^{33}$.

The gauge boson possesses the isospin $SU(3)_I$ and color $SU(3)_C$ symmetries below the grand unification energy. The gauge boson with the energy higher than its mass thus effectively interacts with the Coulomb potential outside the grand unification scale while the gauge boson with the energy lower than its mass essentially interacts with the Yukawa potential within the grand unification or weak scale. At the grand unification scale, the gauge boson has a mass so that the interaction range is limited to the grand unification scale. However, as energy goes down the gauge boson loses mass because of the pseudo-scalar boson condensation. Isospin interactions are represented by both the Coulomb potential and the Yukawa potential with a Fermi weak constant $G_\mathrm{F} \approx 10^{-5}\,\mathrm{GeV}^{-2}$. This implies that the isospin field with the Yukawa interaction does not propagate over the long range due to the heavy mass; recall that the $(V - A)$ current in the weak theory has a point-like interaction with a Fermi coupling constant G_F. At the electroweak phase transition, the isospin interaction range for massive gauge bosons is restricted to the weak scale but the electromagnetic interaction range for the massless photons is infinite. Gauge boson masses change their values at the different energy scale due to the scalar boson condensation. For example, $M_G \approx 10^2$ GeV at the grand unification scale or at the electroweak phase transition scale and $M_G \approx 0.1$ GeV at the QCD cutoff scale. The Fermi coupling constant G_F thus denotes the effective coupling for the Yukawa potential of the isospin field at the electroweak scale. Energy scales for the phase transitions of the fundamental forces are accordingly as follows: the grand unification corresponds to $E \approx \Lambda_H$, the weak force to $E \approx \Lambda_W$ with the weak scale $\Lambda_W \approx 10^2$ GeV, and the strong force to $E \approx \Lambda_\mathrm{QCD}$ with the strong mass scale $\Lambda_\mathrm{QCD} \approx 0.1$ GeV.

5 Quark and lepton mass generation

The DSSB mechanism suggests quark and lepton mass generation, which is the outstanding problem in the GWS model using the Higgs mechanism. In this scheme, quarks and leptons are not treated as fundamental elementary particles but as composite particles. The duality property before phase transition may be broken by DSSB after phase transition.

Ordinary mass terms in the Lagrangian are not allowed, because the left- and right-handed components of the various fermion fields have different quantum numbers, and so simple mass terms violate gauge invariance. To generate masses for the quarks and leptons, the DSSB mechanism of gauge symmetry and chiral symmetry is required. The condensation of pseudo-scalar bosons and fermion pairs is connected with the lepton and quark masses in the course of parity and charge conjugation violation, which breaks chiral symmetry; the $SU(2)_\mathrm{L}$ doublet $(V - A)$ current is conserved but the $SU(2)_\mathrm{R}$ (or $U(1)_\mathrm{R}$) singlet $(V + A)$ current is not conserved. The massless gauge bosons in the DSSB of gauge symmetry and chiral symmetry are photons.

The dual Meissner effect, constituent fermions, fine and hyperfine structure, and quark and lepton mass generation are addressed in the following.

5.1 Dual Meissner effect

The quark or lepton formation is the consequence of the isospin–isospin interaction due to the dual Meissner effect, in which the isotope electric monopole and the isotope magnetic dipole (isospin) are confined inside the quark (or lepton), while the isotope magnetic monopole and the isotope electric dipole are confined in the vacuum. The difference number of right- and left-handed (singlet and doublet) fermions $N_{sd} = N_{ss} - N_{sc}$, the number of left-handed constituent particles N_{ss}, and the right-handed condensation number N_{sc} are introduced.

During the DSSB of gauge symmetry and chiral symmetry, the dual Meissner effect of the isospin electric field in the relativistic case can be described by

$$\partial_\mu \partial^\mu G^\mu = -M_G^2 G^\mu, \quad (40)$$

where the right-hand side is the screening current, $j^\mu_{sc} = -M_G^2 G^\mu$. Recall that the masses of the gauge bosons are expressed by $M_G^2 = M_H^2 - i_f g_i^2 \langle\phi\rangle^2 = i_f g_i^2[\phi^2 - \langle\phi\rangle^2]$ where $\langle\phi\rangle$ represents the condensation of the pseudo-scalar boson and i_f denotes the isospin factor. The screening of the isospin field intensity in the isospin superconducting state is given by

$$\nabla^2 \mathbf{E}_i = M_G^2 \mathbf{E}_i, \quad (41)$$

where $\mathbf{G}$ is the isospin electric field E_i excluded in the vacuum by $\mathbf{E}_i = \mathbf{E}_{i0} \mathrm{e}^{-M_G r}$. Note the difference between the isospin dielectric due to the isospin electric field $\mathbf{E}_i$ and the isospin diamagnetism due to the isospin magnetic field $\mathbf{B}_i$. The mechanism is by analogy connected with the Faraday induction law which opposes the change in the isospin electric flux, rather than the isospin magnetic flux according to Lenz's law.

The gauge boson mass is related to the fermion mass m_f:

$$M_G = \left(\frac{g_{im}^2 |\psi(0)|^2}{m_f}\right)^{1/2} \simeq \sqrt{\pi} m_f i_f \alpha_i \sqrt{N_{sd}}, \quad (42)$$

which is obtained by the analogy of electric superconductivity [31], $M^2 = q^2|\psi(0)|^2/m$: $q = -2e$ and $m = 2m_e$ are replaced by g_{im} and m_f. The isospin magnetic coupling constant, $g_{im} = 2\pi n / i_f^{1/2} g_i = 2\pi N_{sd}^{1/2}/i_f^{1/2} g_i$ by the Dirac quantization condition, is used. $|\psi(0)|^2$ denotes the particle probability density and N_{sd} denotes the difference number of right- and left-handed fermions in intrinsic two-space dimensions.

A fermion mass term in the Dirac Lagrangian has the form $m_f \bar{\psi}\psi = m_f(\bar{\psi}_\mathrm{R}\psi_\mathrm{L} + \bar{\psi}_\mathrm{L}\psi_\mathrm{R})$, where the mass term is equivalent to a helicity flip. Left-handed fermions are put into $SU(2)$ doublets and the right-handed ones into $SU(2)$ singlets. The coherent fermion system is effectively a collection of the Cooper pairs of left- and right-handed fermions, so that the macroscopic occupancy of a single quantum state could occur; all the pairs have the same center of mass momentum known as the coherent state. The fermion mass generation mechanism is the dual pairing mechanism of the constituent fermions, which makes boson-like particles of paired fermions. According to the electric–magnetic duality [28,32,33], the isospin electric flux is quantized by $\Phi_E = \oint \mathbf{E}_i \cdot \mathrm{d}\mathbf{A} = i_f^{1/2} g_i$ in the matter space while the isospin magnetic flux is quantized by $\Phi_B = \oint \mathbf{B}_i \cdot \mathrm{d}\mathbf{A} = g_{im}$ with the isospin magnetic coupling constant g_{im} in the vacuum space: the Dirac quantization condition

$$\sqrt{i_f} g_i g_{im} = 2\pi n = 2\pi \sqrt{N_{sd}} \quad (43)$$

is satisfied with the connection between n and N_{sd}. In the matter space, it is the pairing mechanism of isospin electric monopoles while in the vacuum space, it is the pairing mechanism of isospin magnetic monopoles according to the duality between electricity and magnetism [28, 32,33]: isospin electric monopole pairing and isospin magnetic monopole condensation. In the dual pairing mechanism, the discrete symmetries P, C, T, and CP are dynamically broken. The isospin electric monopole, isospin magnetic dipole, and isospin electric quadrupole remain in the matter space, but the isospin magnetic monopole, isospin electric dipole, and isospin magnetic quadrupole condense in the vacuum space as a consequence of P violation. Antimatter particles condense in the vacuum space while matter particles remain in the matter space as a consequence of C violation: the matter–antimatter asymmetry. The lepton–antilepton asymmetry is also supporting evidence of the discrete symmetry breaking. The electric dipole moment of the neutron and the decay of the neutral kaon decay are the typical examples for T or CP violation.

Normal fermions with the quantum number N_{sd} interact with each other with the isospin symmetric configurations, $c_f^\mathrm{s} = (c_f^b, c_f^n, c_f^z, c_f^f) = (1/3, 1/4, 1/12, 1/16)$, while condensed fermions with the condensation number N_{sc} interact with each other with isospin asymmetric configurations, $c_f^\mathrm{a} = (-2/3, -1/2, -1/6, -1/8)$. Comparing (42) with (16), it is realized that $M_H = \pi^{1/2} m_f i_f \alpha_i N_{ss}^{1/2}$ and $\langle\phi\rangle = (m_f^2 i_f \alpha_i N_{sc}/4)^{1/2}$. Note that (42) is analogous to the fermion mass $m_f = \langle \bar{F}F\rangle / 2\mu^2$ with the condensation of the technifermion F and the extended technicolor scale μ in the extended technicolor model [34]. It is instructive that $m_f = \lambda_f M_G$, where $\lambda_f \simeq 1/(\pi^{1/2} i_f \alpha_i N_{sd}^{1/2})$, depends on the number N_{sd}. The intrinsic quantum number of a constituent fermion is thus important in determining the fermion masses. For example, the u quark with the current quark mass $m_u \approx 5\,\mathrm{MeV}$ has $N_{sd}^u = 10^{10}$ and electron with the mass $m_e \simeq 0.5\,\mathrm{MeV}$ has $N_{sd}^e \simeq 10^{12}$ for the effective coupling constant $G_\mathrm{F} = 10^{-5}\,\mathrm{GeV}^{-2}$ and the gauge boson mass $M_G \simeq 10^2\,\mathrm{GeV}$: $m_e/m_u \simeq 1/10$ and $N_{sd}^e/N_{sd}^u \simeq 1/10^2$. Each quark or lepton holds a different intrinsic quantum number as a distinct state.

5.2 Constituent fermions

Leptons and quarks are postulated to be composite particles consisting of constituent fermions as a result of $U(1)_Y$ gauge theory. This concept is similar to the concept of technicolors or preons [10,23] as more fundamental particles forming quarks and leptons.

The relation between the gauge boson mass and the free fermion mass, which is confirmed in (42), is given by $M_H = \pi^{1/2} m_f i_f \alpha_i N_{ss}^{1/2}$ or

$$M_G = \sqrt{\pi} m_f i_f \alpha_i \sqrt{N_{sd}}, \quad (44)$$

where m_f is the mass of a fermion, N_{sd} is the difference number of left- and right-handed fermions in intrinsic two-space dimensions, and N_{ss} is the number of left-handed fermions. The fermion mass formed as a result of the dual pairing mechanism in the above is composed of constituent particles:

$$m_f = \sum_i^N m_i, \quad (45)$$

where m_i is the constituent particle mass. In the above, N depends on the intrinsic quantum number of the constituent particles: $N = N_{sd}^{3/2}$. For example, $N = 1/L$ with the lepton number L for a constituent particle in the formation of a lepton.

The difference number of fermions N_{sd} is the origin of symmetry violation during DSSB. Fermions with odd parity condense in the vacuum space while fermions with even parity remain in the matter space; for example, magnetic monopoles with odd parity are not observed, but electric monopoles with even parity are observed in the matter space. Discrete symmetries are violated so as to have a complex scattering amplitude and the non-conservation of the right-handed singlet current. This is the main reason for the change of the fermion mass and gauge boson mass.

5.3 Fine and hyperfine structure

In order to obtain the fermion mass formula for fine and hyperfine interactions the analogy of QED is non-relativistically considered to avoid complexity. The fine interactions become isospin–isospin interactions as a result of the $SU(2)_\mathrm{L}$ gauge theory. Hyperfine interactions consist of spin–spin and colorspin–colorspin interactions. The colorspin–colorspin interaction is closely related to the difference between the lepton as a color singlet state and the quark as a color triplet state.

In QED, the dipole moment has the form expected for a Dirac point-like fermion: $\mu_i = (e/2m_e)\sigma_i$ where e is the electric charge of particle, m_e the particle mass, and σ_i its Pauli matrix. The spin–spin interaction due to the magnetic moment leads to the hyperfine splitting of the ground state:

$$\triangle E_\mathrm{hf} = \frac{2}{3}\mu_i \cdot \mu_j |\psi(0)|^2 = \frac{2\pi\alpha_e}{3}\frac{\sigma_i \cdot \sigma_j}{m_i m_j}|\psi(0)|^2, \quad (46)$$

where $\psi(0)$ is the wave function of the two particle system at the origin ($r_{ij} = 0$) and the spin–spin interaction is proportional to $\sigma_i \cdot \sigma_j = 4\mathbf{s}_i \cdot \mathbf{s}_j$. The above result can be taken over to the isospin–isospin interaction as the fine interaction:

$$\triangle E_f = \frac{2\pi g_i^2}{3}\frac{\tau_i \cdot \tau_j}{m_i m_j}|\psi(0)|^2, \quad (47)$$

where g_i is the isospin coupling constant, $i_f = \tau_i \cdot \tau_j$ is the isospin factor, and $\tau_i = 2i$ is the Pauli matrix. The masses m_i and m_j denote constituent fermion masses for each lepton or quark as suggested by the fermion mass formation through the pairing mechanism.

The above result can be taken over to spin–spin and colorspin–colorspin interactions:

$$\triangle E_\mathrm{hf} = \frac{2\pi i_f g_i^2}{3}\frac{\sigma_i \cdot \sigma_j}{m_i m_j}|\psi(0)|^2 + \frac{2\pi i_f g_i^2}{3}\frac{\zeta_i \cdot \zeta_j}{m_i m_j}|\psi(0)|^2, \quad (48)$$

where g_i is the isospin coupling constant, i_f is the symmetric isospin factor, and $\zeta_i = 2c$ is the Pauli matrix. The first term denotes the contribution from the spin–spin interaction and the second term denotes the colorspin–colorspin interaction. The second term in (48) makes the distinction between leptons and quarks and causes quark mixing like the d and s quarks.

5.4 Quark and lepton mass generation

The quark or lepton mass consists of three parts apart from the dual pairing mechanism: constituent particle mass, the fine structure energy, and the hyperfine structure energy.

Combining (45), (47), and (48), the fermion mass formula is thus given by

$$\begin{aligned} m_f = & \sum_i m_i + \frac{2\pi g_i^2}{3}\sum_{i>j}\frac{\tau_i \cdot \tau_j}{m_i m_j}|\psi(0)|^2 \\ & + \frac{2\pi i_f g_i^2}{3}\sum_{i>j}\frac{\sigma_i \cdot \sigma_j}{m_i m_j}|\psi(0)|^2 \\ & + \frac{2\pi i_f g_i^2}{3}\sum_{i>j}\frac{\zeta_i \cdot \zeta_j}{m_i m_j}|\psi(0)|^2, \end{aligned} \quad (49)$$

where $|\psi(0)|^2 \simeq (i_f m_f \alpha_i)^3$ and m_i is the mass of each constituent particle. The first term of the right-hand side denotes the free constituent particle contribution, the second term denotes the isospin–isospin contribution, the third term denotes the spin–spin contribution, and the fourth term denotes the colorspin–colorspin contribution in the mass generation mechanism. The fourth term seems to be the main reason for the mass difference of the lepton as a color singlet state and the quark as a color triplet state. The fundamental particles known as leptons and quarks are postulated to be composite particles with the same third component of spin $\pm 1/2$ and isospin $\pm 1/2$ but a different degeneracy number N_{sd}: N_{sd} is further discussed in the following section. It suggests that the fermion mass generation mechanism is relevant for the $(V + A)$ current anomaly, which reduces the zero-point energy and triggers the DSSB of gauge and chiral symmetries.

Although the GWS model holds many attractive features, the Higgs sector and the fermion mass sector are the least satisfactory aspects of the electroweak theory. The minimal choice of a simple Higgs doublet is sufficient to generate the masses both of the gauge bosons and of fermions, but the masses of the fermions are just parameters of the theory which are not predicted; their empirical values must be taken as input parameters. The GWS model allows the electron to be very light but it cannot explain why the electron is so light compared with the intermediate vector boson. However, this scheme might express a plausible reason for it if the fermion difference number N_{sd} is very large, about 10^{12} order. QWD hints at mass generation through the Θ vacuum and the dual pairing mechanism. QWD produces three generations of fermion families and gauge bosons for the DSSB of the gauge group. In this scheme, neutrinos should have masses as hinted at by [35,36]. QWD as a gauge theory has the favorable feature of only one input parameter for the weak coupling constant g_i; the dynamics can be regarded as quantum flavordynamics for fermion generations. Being anomaly free in perturbative renormalization, which requires equal numbers in quarks and leptons, suggests that the lepton number excess compared to the baryon number might be used for the dark matter, which plays an important role in the later stages of the evolution of the universe. The presence of the non-perturbative anomaly, the Θ vacuum term in (6), does not necessarily spoil renormalization because there is no Ward–Takahashi identity [37] destroyed by the non-conservation of any local conserved current with even parity. QWD as the $SU(3)_I$ gauge theory suggests the possibility not for the separate conservation of baryon number and lepton number but for the combined conservation of the $(B-L)$ number in the energies above 10^2 GeV. The lepton number is however conserved as a result of the $U(1)_Y$ gauge theory below the weak scale, and the baryon number is conserved as a result of the $U(1)_Z$ gauge theory below the strong scale [6, 7].

6 Θ constant and quantum numbers

The Θ term in the Lagrangian density triggers DSSB and quantizes space and time. The parameter Θ is constrained to hold the flat universe condition. The gauge invariance and boundary condition of spacetime provide the quantization of the internal space and the external space. The Θ constant in (10) is related to the decay of the neutral kaon, and is applied to the mechanism of fermion mass generation. In this section, the Θ constant, matter and vacuum quantum numbers, and Θ constant and quantum numbers are addressed.

6.1 Θ constant

Under the constraint of the extremely flat universe required by quantum gauge theory, the Θ constant in (6), $\Theta = 10^{-61}\rho_G/\rho_{\mathrm{m}}$, depends on the gauge boson mass M_G since $\rho_G = M_G^4$: $\Theta = 10^{-61}M_G^4/\rho_c \simeq 10^{-4}$ at the weak scale $M_G \simeq 10^2$ GeV. Since the gauge boson mass depends on the Weinberg mixing angle θ_{W}, $M_G' \to M_G \sin\theta_{\mathrm{W}}$, during the DSSB of the $SU(3)_I \to SU(2)_{\mathrm{L}} \times U(1)_Y$ or the $SU(2)_{\mathrm{L}} \times U(1)_Y \to U(1)_e$ gauge theory, the change of the Θ constant depends on θ_{W}: $\Delta\Theta \propto \sin^4\theta_{\mathrm{W}} = i_f^{e2}$. Note that the isospin factor $i_f^w = \sin^2\theta_{\mathrm{W}} \simeq 1/4$ and the weak boson mass $M_W = M_Z \cos\theta_{\mathrm{W}}$. The relation between the Θ constant and the difference number N_{sd} is given by

$$\Theta = \pi^2 i_f^4 \alpha_i^4 m_f^4 N_{sd}^2/10^{61}\rho_c \qquad (50)$$

from (10) and (44). Θ values become $\Theta_{\mathrm{EW}} \approx 10^{-4}$ and $\Theta_{\mathrm{QCD}} \approx 10^{-13}$ at different stages. This is consistent with the observed results, $\Theta < 10^{-9}$ in the electric dipole moment of the neutron and $\Theta \simeq 10^{-3}$ in the neutral kaon decay.

6.2 Matter and vacuum quantum numbers

There is a condensation process in the fermion mass generation mechanism. This process is the dual pairing mechanism of constituent fermions, which makes boson-like particles of paired fermions. At the phase transition, N_{sc} becomes zero so that N_{sd} becomes the maximum. Using the relations $M_G = \pi^{1/2} m_f i_f \alpha_i (N_{sd}^{1/2})$ and $M_G^2 = M_H^2 - i_f g_i^2 \langle\phi\rangle^2 = i_f g_i^2[\phi^2 - \langle\phi\rangle^2]$, the zero-point energy $M_H^2 = \pi m_f^2 i_f^2 \alpha_i^2 N_{ss}$ and the reduction of the zero-point energy $\langle\phi\rangle^2 = m_f^2 i_f \alpha_i N_{sc}/4$ are obtained. The difference number of right–left-handed singlet fermions N_{sd} in intrinsic two-space dimensions suggests the introduction of a degenerate particle number N_{sp} in the intrinsic radial coordinate and an intrinsic principal number n_m; particle quantum numbers are related by $n_m^4 = N_{sp}^2 = N_{sd}$ and the Dirac quantization condition $i_f^{1/2} g_i g_{im} = 2\pi N_{sp}$ is satisfied. The N_{sp} is thus the degenerate state number in the intrinsic radial coordinate that has the same principal number n_m. The intrinsic principal quantum number n_m consists of three quantum numbers, that is, $n_m = (n_c, n_i, n_s)$, where n_c is the intrinsic principal quantum number for the color space, n_i is the intrinsic principal quantum number for the isospin space, and n_s is the intrinsic principal quantum number for the spin space. The intrinsic quantum numbers (n_c, n_i, n_s) take integer numbers. A fermion therefore possesses a set of intrinsic quantum numbers (n_c, n_i, n_s) to represent its intrinsic quantum states.

This concept automatically adopts the three types of intrinsic angular momentum operators, $\hat{C}$, $\hat{I}$, and $\hat{S}$, when the intrinsic potentials for color, isospin, and spin charges are central so that they depend on the intrinsic radial distance: for instance, the color potential in the strong interactions is dependent on the radial distance. The intrinsic spin operator $\hat{S}$ has a magnitude squared $\langle S^2\rangle = s(s+1)$ and $s = 0, 1/2, 1, 3/2, \cdots, (n_s - 1)$. The third component of $\hat{S}$, $\hat{S}_z$, has half integer or integer quantum number in the range of $-s \sim s$ with the degeneracy $2s+1$. The intrinsic isospin operator $\hat{I}$ analogously has a magnitude squared $\langle I^2\rangle = i(i+1)$ and $i = 0, 1/2, 1, 3/2, \cdots, (n_i - 1)$. The third component of $\hat{I}$, $\hat{I}_z$, has half integer or integer quantum number in the range of $-i \sim i$ with the degeneracy $2i+1$. The intrinsic color operator $\hat{C}$ analogously has a magnitude squared $\langle C^2\rangle = c(c+1)$ and $c = 0, 1/2, 1, 3/2, \cdots, (n_c - 1)$. The third component of $\hat{C}$, $\hat{C}_z$, has half integer or integer quantum number in the range of $-c \sim c$ with the degeneracy $2c+1$. The principal number n_m in intrinsic space quantization is very much analogous to the principal number n in extrinsic space quantization and the intrinsic angular momenta are analogous to the extrinsic angular momentum so that the total angular momentum has the form of $\mathbf{J} = \mathbf{L} + \mathbf{S} + \mathbf{I} + \mathbf{C}$, which is the extension of the conventional total angular momentum $\mathbf{J} = \mathbf{L} + \mathbf{S}$. The intrinsic principal number n_m denotes the intrinsic spatial dimension or radial quantization: $n_c = 3$ represents the strong interactions as an $SU(3)_C$ gauge theory and $n_i = 3$ represents the weak interactions as an $SU(3)_I$ gauge theory. For QWD as the $SU(3)_I$ gauge theory, there are nine weak gauge bosons ($n_i^2 = 3^2 = 9$), which consist of one singlet gauge boson G_0 with $i = 0$, three degenerate gauge bosons $G_1 \sim G_3$ with $i = 1$, and five degenerate gauge bosons $G_4 \sim G_8$ with $i = 2$; for the GWS model as the $SU(2)_{\mathrm{L}} \times U(1)_Y$ gauge theory, one singlet gauge boson G_0 with $i = 0$, three gauge bosons $G_1 \sim G_3$ with $i = 1$, and one gauge boson G_8 with $i = 2$ are required. One explicit piece of evidence of colorspin and isospin angular momenta is the strong isospin symmetry in nucleons, which is postulated to be the combination symmetry of colorspin and weak isospin in this scheme. Other evidence is the nuclear magnetic dipole moment: the Landé spin g-factors of the proton and neutron are respectively $g_s^p = 5.59$ and $g_s^n = -3.83$, which are shifted from 2 and 0, because of contributions from color and isospin degrees of freedom as well as spin degrees of freedom. The mass ratio of the proton and the constituent quark, $m_p/m_q \sim 2.79$, thus represents three intrinsic degrees of freedom of color, isospin, and spin. In fact, the extrinsic angular momentum associated with the intrinsic angular momentum may be decomposed by $\mathbf{L} = \mathbf{L}_i + \mathbf{L}_c + \mathbf{L}_s$, where $\mathbf{L}_i$ is the angular momentum originating from the isospin charge, $\mathbf{L}_c$ is the angular momentum originating from the color charge, and $\mathbf{L}_s$ is the angular momentum originating from the spin charge. This is supported by the fact that the orbital angular momentum l_c of a nucleon has a different origin from the color charges with the orbital angular momentum l_i of the electron from the isospin charge, since two angular momenta have opposite directions from the information of the spin–orbit couplings in nucleus and atoms. Extrinsic angular momenta have extrinsic parity $(-1)^l = (-1)^{(l_c+l_i+l_s)}$, intrinsic angular momenta have intrinsic parity $(-1)^{(c+i+s)}$, and the total parity becomes $(-1)^{(l+c+i+s)}$ for the electric moments; while the extrinsic angular momenta have extrinsic parity $(-1)^{(l+1)} = (-1)^{(l_c+l_i+l_s+1)}$, the intrinsic angular momenta have intrinsic parity $(-1)^{(c+i+s+1)}$, and the total parity becomes $(-1)^{(l+c+i+s+1)}$ for the magnetic moments.

Fermions increase their masses by decreasing their intrinsic principal quantum numbers from the higher ones at higher energies to the lower ones at lower energies. The coupling constant α_i of a non-Abelian gauge theory is strong for the small N_{sd} and is weak for the large N_{sd} according to the renormalization group analysis. The vacuum energy is described by the zero-point energy in units of $\omega/2$ with the maximum number $N_{sd} \simeq 10^{61}$ and the vacuum is filled with fermion pairs of up and down colorspins, isospins, or spins, whose pairs behave like bosons quantized by the unit of ω: this is analogous to the superconducting state of fermion pairs. The intrinsic particle number $N_{sp} \simeq 10^{30}$ (or $B \simeq 10^{-12}$, $L \simeq 10^{-9}$) characterizes gravitational interactions for fermions with the mass $10^{-12}\,\mathrm{GeV}$, $N_{sp} \simeq 10^6$ (or $L_e \simeq 1$) characterizes the weak interactions for electrons, and $N_{sp} \simeq 1$ (or $B \simeq 1$) characterizes the strong interactions for nucleons. The fundamental particles known as leptons and quarks are hence postulated to be composite particles with the color, isospin, and spin quantum numbers; the quark is a color triplet state, but the lepton is a color singlet state. Note that if $N_{sp} > 1$ (or $B < 1$), it represents a point-like fermion and if $N_{sp} < 1$ (or $B > 1$), it represents a composite fermion. There is accordingly the possibility of internal structure for the lepton or quark when they are considered as composite particles. Since the Θ constant is thus parameterized by $\Theta = 10^{-61}\rho_G/\rho_{\mathrm{m}}$ with the vacuum energy density $\rho_G = M_G^4$ and the matter energy density $\rho_{\mathrm{m}} \simeq \rho_c \simeq 10^{-47}\,\mathrm{GeV}^4$, the relation between the Θ constant and the difference number N_{sd} is given by $\Theta = \pi^2 m_f^4 i_f^4 \alpha_i^4 N_{sd}^2/10^{61}\rho_c$.

The weak vacuum represented by massive gauge bosons is quantized by the maximum wavevector mode $N_R \approx 10^{30}$, the total gauge boson number $N_G = 4\pi N_R^3/3 \approx 10^{91}$, and the gauge boson number density $n_G = \Lambda_{\mathrm{EW}}^3 \approx 10^{47}\,\mathrm{cm}^{-3}$. Baryon matter represented by massive baryons is spatially quantized by the maximum wavevector mode $N_F \approx 10^{26}$ and the total baryon number $B = N_B = 4\pi N_F^3/3 \approx 10^{78}$ [7,19]. The baryon matter quantization described above is consistent with the nuclear matter density $n_n \approx n_B \approx 1.95 \times 10^{38}\,\mathrm{cm}^{-3}$ and Avogadro's number $N_A = 6.02 \times 10^{23}\,\mathrm{mol}^{-1} \approx 10^{19}\,\mathrm{cm}^{-3}$ in the matter. Electrons with the mass $0.5\,\mathrm{MeV}$ might be similarly quantized by $N_F \approx 10^{27}$ and the total number 10^{81} if the electron number is conserved under the assumption of $\Omega_e = \rho_e/\rho_c \approx 1$: since the conservation of the baryon number minus the lepton number $(B-L)$ as well as the baryon number (B) and the lepton number (L) is a good quantum number at low energies, the total electron number 10^{81}, different from the total baryon number 10^{78}, suggests lepton matter as dark matter. The maximum wavevector mode N_F is close to 10^{30} if the quantization unit of fermions $10^{-12}\,\mathrm{GeV}$ (the baryon number $B \simeq 10^{-12}$) is used rather than the unit of baryons, $0.94\,\mathrm{GeV}$ (the baryon number $B = 1$) under the assumption of the fermion number conservation $N_f \simeq 10^{91}$.

6.3 Θ constant and quantum numbers

The invariance of the gauge transformation provides us with $\psi[\hat{O}_\nu] = \mathrm{e}^{\mathrm{i}\nu\Theta}\psi[\hat{O}]$ for the fermion wave function ψ with the transformation of an operator $\hat{O}$ by the class ν gauge transformation, $\hat{O}_\nu$: the vacuum state characterized by the constant Θ is called the Θ vacuum [13]. The true vacuum is the superposition of all the $|\nu\rangle$ vacua with the phase $\mathrm{e}^{\mathrm{i}\nu\Theta}$: $|\Theta\rangle = \sum_\nu \mathrm{e}^{\mathrm{i}\nu\Theta}|\nu\rangle$. The topological winding number ν or the topological charge q_s is defined by

$$\nu = \nu_+ - \nu_- = \int \frac{i_f g_i^2}{16\pi^2} \mathrm{Tr} F^{\mu\nu} \tilde{F}_{\mu\nu} \mathrm{d}^4 x, \qquad (51)$$

where the subscripts + and − denote moving particles with chiralities + and − respectively in the presence of the gauge fields [38]. The matter energy density generated by the surface effect is postulated to be

$$\rho_m \simeq \rho_c \simeq \frac{i_f g_i^2}{16\pi^2}\mathrm{Tr}F^{\mu\nu}\tilde{F}_{\mu\nu}, \quad (52)$$

which implies that the fermion mass is generated by the difference of the fermion numbers moving to left- and right-handed directions. In this respect, the difference number N_{sd}, the left-handed fermion number N_{ss}, and the condensed right-handed fermion number N_{sc} in intrinsic two-space dimensions respectively correspond to ν, ν_+, and ν_- in three space and one time dimensions. In the presence of the Θ term, the singlet (V+A) current is not conserved due to an Adler–Bell–Jackiw anomaly [12]: $\partial_\mu J_\mu^{1+\gamma^5} = (N_f i_f g_i^2/16\pi^2)\mathrm{Tr}F^{\mu\nu}\tilde{F}_{\mu\nu}$, with the flavor number of fermions N_f, and this reflects degenerate multiple vacua. This illustrates mass generation by the surface effect due to the field configurations with parallel isospin electric and magnetic fields. If $\nu = \rho_\mathrm{m}/\rho_G$ is introduced from (51) and (52), the condition $\Theta\nu = 10^{-61}$ is satisfied. The Θ value parameterized by $\Theta = 10^{-61}\rho_G/\rho_\mathrm{m}$ is consistent with the observed result, $\Theta \simeq 10^{-3}$, in neutral kaon decay [16].

The topological winding number ν is related to the intrinsic quantum number n_m by $\nu = n_m^{-8}$. The intrinsic principal number n_m is also connected with N_{sp} and N_{sd}: $n_m^2 = N_{sp}$, $N_{sp}^2 = N_{sd}$, and $N_{sp}^4 = 1/\nu$. The relation between the intrinsic radius and the intrinsic quantum number might be written $r_i = r_{0i}/n_m^2$, with the radius $r_{0i} = 1/2m_f\alpha_z = N_{sp}/M_G$. The intrinsic quantum numbers are exactly analogous to the extrinsic quantum numbers. The extrinsic principal number n for the nucleon is related to the nuclear mass number A or the baryon quantum number B: $n^2 = A^{1/3}$, $n^4 = A^{2/3}$, $n^6 = B = A$. The relation between the nuclear radius and the extrinsic quantum number is outlined by

$$r = r_0 A^{1/3} = r_0 n^2, \quad (53)$$

with the radius $r_0 = 1.2\,\mathrm{fm}$ and the nuclear principal number n. This is analogous to the atomic radius $r_e = r_0 n_e^2$ with the atomic radius r_0 and the electric principal number n_e: the atomic radius $r_0 = 1/2m_e\alpha_y$ is almost the same as the Bohr radius $a_B = 1/m_e\alpha_e = 0.5\times10^{-8}\,\mathrm{cm}$. These concepts are related to the constant nuclear density $n_B = 3/4\pi r_0^3 = 1.95\times10^{38}\,\mathrm{cm}^{-3}$ or Avogadro's number $N_A = 6.02\times10^{23}\,\mathrm{mol}^{-1}$ and to the constant electron density $n_e = 3/4\pi r_e^3 = 6.02\times10^{23} Z\rho_\mathrm{m}/A$ with the nuclear mass density ρ_m in units of g/cm^3, where the possible relation is

$$r_e = r_0 L^{1/3} = r_0 n_e^2, \quad (54)$$

with the lepton number L.

The Θ values according to (10) become $\Theta_\mathrm{Pl} \approx 10^{61}$, $\Theta_\mathrm{EW} \approx 10^{-4}$, $\Theta_\mathrm{QCD} \approx 10^{-13}$, and $\Theta_0 \approx 10^{-61}$ at different stages. The scope of $\Theta = 10^{61} \sim 10^{-61}$ corresponds to the scope of $\nu = 10^{-122} \sim 10^0$ to satisfy the flat universe condition $\nu\Theta = 10^{-61}$: the maximum quantization number $N_{sp} \simeq N_R \simeq 10^{30}$ and $N_G \simeq 4\pi N_R^3/3 \simeq 10^{91}$. The maximum wavevector mode $N_R = (\rho_G/\Theta\rho_\mathrm{m})^{1/2} = 10^{30}$ of the weak vacuum is obtained. These describe possible dualities between intrinsic quantum numbers and extrinsic quantum numbers: n_m and n, N_{sp}^3 and A, and $1/\nu$ and $A^{4/3}$ for baryons. The Θ term as the surface term modifies the original GWS model [1] for the weak interactions, which has a problem in the fermion mass violating gauge invariance, and this suggests mass generation as the non-perturbative breaking of gauge and chiral invariance through DSSB.

7 Comparison among quantum weakdynamics, Glashow–Weinberg–Salam model, and grand unified theory

This section is devoted to a summary, and to show QWD extending beyond the SM or toward a new GUT as an $SU(3)_C\times SU(3)_I$ gauge theory. Table 3 summarizes a comparison among QWD, GWS model, and GUT. GUT in the table represents $SU(5)$ gauge theory or $SO(10)$ gauge theory [3].

There are many unexplained phenomena in the SM. For instances, there are no clues in the SM for many free parameters, three family generations for the leptons and quarks, matter mass generation, the Higgs problem or vacuum problem, dynamical spontaneous symmetry breaking (DSSB) beyond spontaneous symmetry breaking, the neutrino mass problem, etc. In order to resolve these problems, grand unified theories (GUTs) were proposed [3]. Nevertheless, the grand unification of strong and electroweak interactions is not complete and GUTs also have model dependent problems: the hierarchy problem, proton decay, and the Weinberg angle are problems in $SU(5)$ gauge theory [3].

Already confirmed predictions of QWD generating the GWS model are discrete symmetry breaking, $V - A$ current conservation, massive gauge bosons, the Fermi weak coupling constant G_F, the Weinberg angle $\sin^2\theta_\mathrm{W} = 1/4$, the Cabbibo angle $\sin\theta_\mathrm{C} = 1/4$, and the three generations of leptons and quarks as discussed in Sect. 3.

New predictions from QWD beyond the GWS model, which are to be tested by experiments, are as follows: the existence of isospin 3/2 leptons or quarks in addition to isospin 1/2 fermions, the eight-fold way of isospin 1/2 fermions as composite particles, CP violation with $\Theta \simeq 10^{-4}$ order, the existence of massive gauge bosons $G_4 \sim G_7$ with isospin 2 in addition to gauge bosons $G_1 \sim G_3$ with isospin 1, the $(B-L)$ current conservation above the weak scale, the new unification scale of the strong and weak forces at the order of $10^3\,\mathrm{GeV}$, fermion mass generation, neutrino mass and oscillation confirmed by recent experiments, quark flavor mixing due to color charge, the isospin coupling constant hierarchy (α_i, $\alpha_z = \alpha_i/3$, $\alpha_\mathrm{w} = \alpha_\mathrm{w}/4, \alpha_y = \alpha_i/12$, and $\alpha_e = \alpha_i/16$), the lepton asymmetry in the universe, neutron–antineutron oscilla-

Table 3. Comparison among quantum weakdynamics, Glashow–Weinberg–Salam model, and grand unified theory

Classification	QWD	GWS	GUT
Grand unification energy	10^3 GeV		10^{15} GeV
Symmetry breaking	DSSB	SSB	SSB
Discrete symmetries (P, C, T, CP)	breaking	breaking	breaking
Θ vacuum	yes	no	no
Higgs bosons	no	yes	yes
Lepton number conservation	yes	yes	no
Electron number conservation	yes ($N_e \simeq 10^{81}$)	yes	yes
Baryon number conservation	no	no	no
Proton decay	unknown	no	yes
Electron decay	unknown	no	unknown
Weinberg angle	$\sin^2\theta_{\mathrm{W}} = 1/4$	free parameter	$\sin^2\theta_{\mathrm{W}} = 3/8$
Cabbibo angle	$\sin\theta_{\mathrm{C}} = 1/4$	free parameter	free parameter
Fermion mass generation	yes	unsatisfactory	unsatisfactory
Coupling constant hierarchy	yes	no	no
Neutrino mass	yes	no	unknown
Number of gauge bosons	9	4	24
Free parameters	coupling constant	many	several

tions, the analogy between QWD and QCD as $SU(3)$ gauge theories, etc.

8 Conclusions

This study proposes that a certain group H leads to an $SU(3)_I \times SU(3)_C$ group for weak and strong interactions at a grand unification scale through dynamical spontaneous symmetry breaking (DSSB); the group chain is $H \supset SU(3)_I \times SU(3)_C$. The grand unification of the $SU(3)_I \times SU(3)_C$ group beyond the standard model with the group $SU(3)_C \times SU(2)_{\mathrm{L}} \times U(1)_Y$ provides the coupling constant $\alpha_i = \alpha_{\mathrm{s}} \simeq 0.12$ at a new grand unification scale around 10^3 GeV, which might be the resolution to the hierarchy problem of the grand unification scale 10^{15} GeV. DSSB consists of two simultaneous mechanisms; the first mechanism is the explicit symmetry breaking of gauge symmetry, which is represented by the isospin (isotope) factor i_f and the weak coupling constant g_i, and the second mechanism is the spontaneous symmetry breaking of gauge fields, which is represented by the condensation of the pseudo-scalar fields postulated as spatially longitudinal components of the gauge fields. At the energy $T \simeq 10^2$ GeV, the gauge boson number density $n_G = M_G^3 \approx 10^6\,\mathrm{GeV}^3 \approx 10^{47}\,\mathrm{cm}^{-3}$, the vacuum energy density $V_0 = M_G^4 \approx 10^8\,\mathrm{GeV}^4 \approx 10^{25}\,\mathrm{gcm}^{-3}$, and the total gauge boson number $N_G \approx 10^{91}$ are predicted.

Quantum weakdynamics (QWD) as an $SU(3)_I$ gauge theory predicts the free parameters in the GWS model being an $SU(2)_{\mathrm{L}} \times U(1)_Y$ gauge theory. QWD provides the Weinberg angle, the Cabbibo angle, quark and lepton families, a modification of the Higgs mechanism, fermion mass generation, etc. QWD is dynamically spontaneous symmetry broken through the condensation of pseudo-scalar bosons. QWD generates electroweak theory as an $SU(2)_{\mathrm{L}} \times U(1)_Y$ gauge theory at the electroweak scale and then electroweak theory produces QED as a $U(1)_e$ gauge theory through DSSB. The essential point is that the DSSB mechanism is adapted to all the interactions characterized by gauge invariance, the physical vacuum problem, and discrete symmetry breaking. This work suggests that the electroweak interactions originate from the $SU(3)_I$ gauge theory for the weak force, provided that scalar bosons play the roles of Higgs particles producing DSSB. The effective coupling constant chain due to massive gauge bosons is $G_H \supset G_{\mathrm{F}} \times G_R$ for grand unification, weak, and strong interactions respectively. Quark and lepton families seem to be successfully generated in terms of the mixing of the $SU(2)$ three subgroups in two octets. The DSSB of local gauge symmetry and global chiral symmetry triggers the $(V + A)$ current anomaly. The $(V - A)$ current conservation and the $(V + A)$ current non-conservation are thus explained and the absence of right-handed neutrinos is a result of the $(V + A)$ current non-conservation. The electroweak coupling constants derived in terms of the Weinberg angle $\sin^2\theta_{\mathrm{W}} = 1/4$ are $\alpha_z = i_f^z\alpha_i = \alpha_i/3 \simeq 0.04$, $\alpha_{\mathrm{w}} = i_f^w\alpha_i = \alpha_i/4 \simeq 0.03$, $\alpha_y = i_f^y\alpha_i = \alpha_i/12 \simeq 0.01$, and $\alpha_e = i_f^e\alpha_i = \alpha_i/16 \simeq 1/133$ for symmetric isospin interactions at the weak scale and $-2\alpha_i/3$, $-\alpha_i/2$, $-\alpha_i/6$, and $-\alpha_i/8$ for asymmetric isospin interactions. The isospin factors are $i_f^{\mathrm{s}} = (i_f^z, i_f^w, i_f^y, i_f^e) = (1/3, 1/4, 1/12, 1/16)$ for symmetric interactions and $i_f^{\mathrm{a}} = (-2/3, -1/2, -1/6, -1/8)$ for asymmetric interactions: $i_f^w = \sin^2\theta_{\mathrm{W}}$ and $i_f^e = \sin^4\theta_{\mathrm{W}}$. The symmetric charge factors reflect an intrinsic even parity with repulsive force while the asymmetric charge factors reflect an intrinsic odd parity with attractive force; this suggests electromagnetic duality. The Cabbibo angle $\sin\theta_{\mathrm{C}} \simeq 1/4$ predicts that two more massive gauge bosons with $M_G = 2M_W$ in addition to four intermediate vector bosons exist at the weak scale for symmetric isospin interactions and quark flavor mixing is due to color degrees of freedom for quarks. QWD for weak interactions is also proposed as the analogous dynamics of QCD for strong interactions. Common characteristics of gauge theories, such as gauge symmetry, the true vacuum problem, and discrete symmetry breaking for both weak and strong interactions, are understood in terms of the concepts of the analogy property and the DSSB mechanism.

The mechanism of fermion mass generation is suggested in terms of the DSSB of gauge symmetry and chiral symmetry known as the dual pairing mechanism of the superconducting state: $M_G = \pi^{1/2} m_f i_f \alpha_i N_{sd}^{1/2}$ with the difference number of right–left-handed fermions N_{sd} in intrinsic two-space dimensions. The Θ constant is parameterized by $\Theta = 10^{-61}\rho_G/\rho_{\rm m}$ with the vacuum energy density $\rho_G = M_G^4$ and the matter energy density $\rho_{\rm m}$. N_{sd} suggests the introduction of a degenerate particle number N_{sp} in the intrinsic radial coordinate and an intrinsic principal number n_m; particle numbers are linked to the relation $n_m^4 = N_{sp}^2 = N_{sd}$ and the Dirac quantization condition $i_f^{1/2} g_i g_{im} = 2\pi N_{sp}$ is satisfied. The intrinsic principal quantum number n_m consists of three quantum numbers, that is, $n_m = (n_c, n_i, n_s)$ where n_c is the intrinsic principal quantum number for the color space, n_i is the intrinsic principal quantum number for the isospin space and n_s is the intrinsic principal quantum number for the spin space. This concept automatically adopts the three types of intrinsic angular momentum operators, $\hat{C}$, $\hat{I}$, and $\hat{S}$, when the intrinsic potentials for color, isospin, and spin charges are central so that they depend on the intrinsic radial distance. The principal number n_m in intrinsic space quantization is very much analogous to the principal number n in extrinsic space quantization and the intrinsic angular momenta are analogous to the extrinsic angular momentum so that the total angular momentum has the form of $\mathbf{J} = \mathbf{L} + \mathbf{S} + \mathbf{I} + \mathbf{C}$, which is the extension of the conventional total angular momentum $\mathbf{J} = \mathbf{L} + \mathbf{S}$. The intrinsic particle number $N_{sp} \simeq 10^6$ (or $L_e \simeq 1$) characterizes weak interactions for electrons, and $N_{sp} \simeq 1$ (or $B \simeq 1$) characterizes strong interactions for nucleons. Fundamental particles known as leptons and quarks are hence postulated as composite particles with the color, isospin, and spin quantum numbers; the quark is a color triplet state but the lepton is a color singlet state. If $N_{sp} > 1$ (or $B < 1$), it represents a point-like fermion and if $N_{sp} < 1$ (or $B > 1$), it represents a composite fermion. The Θ value defined by $\Theta = 10^{-61}\rho_G/\rho_{\rm m}$ is consistent with the observed result, $\Theta \simeq 10^{-3}$, in neutral kaon decay. The fact that a gauge theory allows one to generate the masses of fermions and gauge bosons without spoiling the renormalizability is very important; the renormalizability of a gauge theory with spontaneous symmetry breaking was demonstrated by 't Hooft [39].

In this scheme, the vacuum and matter energies are spatially quantized as well as the photon energy. The weak vacuum represented by massive gauge bosons is quantized by the maximum wavevector mode $N_R \approx 10^{30}$, the total gauge boson number $N_G = 4\pi N_R^3/3 \approx 10^{91}$, and the gauge boson number density $n_G = \Lambda_{\rm EW}^3 \approx 10^{47}\,{\rm cm}^{-3}$. The baryon matter represented by massive baryons is quantized by the maximum wavevector mode (Fermi mode) $N_F \approx 10^{26}$ and the total baryon number $B = N_B = 4\pi N_F^3/3 \approx 10^{78}$. Electrons might be similarly quantized by the maximum wavevector mode $N_F \approx 10^{27}$ and the total particle number $L_e = N_e \approx 10^{81}$ for electrons with the mass 0.5 MeV. The maximum wavevector mode N_F is close to 10^{30} if the quantization unit of the fermions 10^{-12} GeV is used rather than the unit of baryons 0.94 GeV under the assumption of the fermion number conservation. The baryon matter quantization described above is consistent with the nuclear matter density $n_n \approx n_B \approx 1.95 \times 10^{38}\,{\rm cm}^{-3}$ and Avogadro's number $N_A = 6.02 \times 10^{23}\,{\rm mol}^{-1} \approx 10^{19}\,{\rm cm}^{-3}$ in the matter. Massless photons are quantized by the maximum wavevector mode $N_\gamma \approx 10^{29}$ and the total photon number $N_{t\gamma} = 4\pi N_\gamma^3/3 \approx 10^{88}$. These total particle numbers $N_G \approx 10^{91}$, $N_B \approx 10^{78}$, and $N_{t\gamma} \approx 10^{88}$ are conserved good quantum numbers. The quantization unit of vacuum energy due to a gauge boson in the weak interactions is $\Lambda_{\rm EW}/N_R \simeq 10^{-28}$ GeV.

The notable accomplishments of this work are summarized in the following. The grand unification of the $SU(3)_I \times SU(3)_C$ gauge theory beyond the standard model of the $SU(3)_C \times SU(2)_{\rm L} \times U(1)_Y$ gauge theory is suggested in the scale around 10^3 GeV rather than in the conventional unification scale 10^{15} GeV. This seems to be a viable solution of the hierarchy problem. The unification of the $SU(2)_{\rm L} \times U(1)_Y$ electroweak theory, the GWS model, is developed in terms of the $SU(3)_I$ gauge theory, QWD, if scalar bosons postulated to be spatially longitudinal components of the gauge bosons play the roles of Higgs particles during DSSB. The DSSB of local gauge symmetry and global chiral symmetry gives rise to the $(V + A)$ current anomaly. The quark and lepton families are successfully generated from two octets of triplet isospin combinations, and the predicted Weinberg angle and Cabbibo angle are in good agreement with the experimental values. Photons are regarded as massless gauge bosons indicating DSSB and the fine structure constant in the electromagnetic interactions is related to the fine structure constant in weak interactions, $\alpha_e = \alpha_i/16$. Fermion mass generation is suggested in terms of the DSSB of gauge symmetry and chiral symmetry due to the $(V + A)$ current anomaly. The baryon minus lepton number $(B-L)$ conservation is a consequence of the $SU(3)_I$ gauge theory, the lepton number conservation is a consequence of the $U(1)_Y$ gauge theory, and the electron number conservation is a consequence of the $U(1)_e$ gauge theory. The development of this theory would thus shed light on understanding the fundamental forces in nature and its consequences play significant roles in various fields since point-like particles like quarks and leptons are more or less governed by the electroweak force explained by QWD.

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